Breaking Local Symmetries in Quasigroup Completion Problems

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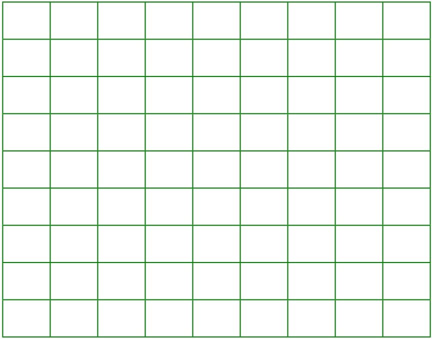
Outline

- Motivation
- Quasigroup Completion Problems (QCPs)
- Symmetry Breaking in QCPs
- Experimental Results
- Conclusions & Future Work

Motivation (Symmetry)



Motivation (Symmetry)

					1			

Motivation (Symmetry)

4	8	7	5	1	2	9	3	6

Motivation (Symmetry)

		3						
		7						
		5						
		2						
		8						
		1						
		9						
		6						
		4						

Motivation (Symmetry)

6	9	3	7	8	4	5	1	2
4	8	7	5	1	2	9	3	6
1	2	5	9	6	3	8	7	4
9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
3	1	9	4	7	5	2	6	8
8	5	6	1	2	9	7	4	3
2	7	4	8	3	6	1	5	9

Motivation (Symmetry)

6	9	3	7	8	4	5	1	2
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Motivation (Symmetry)

4	8	7	5	1	2	9	3	6
6	9	3	7	8	4	5	1	2
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9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
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Motivation (Symmetry)

6	7	3	9	8	4	5	1	2
4	5	7	8	1	2	9	3	6
1	9	5	2	6	3	8	7	4
9	6	2	3	5	1	4	8	7
5	2	8	6	4	7	3	9	1
7	3	1	4	9	8	6	2	5
3	4	9	1	7	5	2	6	8
8	1	6	5	2	9	7	4	3
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Motivation (Quasigroup Completion Problem)

							1	
4								
	2							
				5		4		7
		8						
		1		9				
3			4					
	5		1					
			8		6			

Motivation (Quasigroup Completion Problem)

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5	6	8	2	4	7	3	9	1
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3	1	9	4	7	5	2	6	8
8	5	6	1	2	9	7	4	3
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4	8	7	5	1	2	9	3	6
1	2	5	9	6	3	8	7	4
9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
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Quasigroup Completion Problems (QCPs)

Definition (Quasigroup)

- A **quasigroup** is an ordered pair $(Q, \cdot)$, where Q is a set and $\cdot$ is a binary operation on Q such that for each a and b in Q, there exist unique elements x and y in Q such that:
  1. $a \cdot x = b$,
  2. $y \cdot a = b$.

  The **order** n of the quasigroup is the cardinality of the set Q.

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  1. $a \cdot x = b$,
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  The **order** n of the quasigroup is the cardinality of the set Q.

Definition (Latin Square)

- A **Latin square** is an $n \times n$ table filled with n different symbols, having one symbol in each cell, in such a way that each symbol occurs exactly once in each row and exactly once in each column.

Quasigroup Completion Problems (QCPs)

·	1	2	3	4	5	6	7	8	9
1	6	9	3	7	8	4	5	1	2
2	4	8	7	5	1	2	9	3	6
3	1	2	5	9	6	3	8	7	4
4	9	3	2	6	5	1	4	8	7
5	5	6	8	2	4	7	3	9	1
6	7	4	1	3	9	8	6	2	5
7	3	1	9	4	7	5	2	6	8
8	8	5	6	1	2	9	7	4	3
9	2	7	4	8	3	6	1	5	9

- Latin square as the multiplication table of a quasigroup:
  - Unique elements x and y in Q such that: $a \cdot x = b$ and $y \cdot a = b$.

Quasigroup Completion Problems (QCPs)

·	1	2	3	4	5	6	7	8	9
1	6	9	3	7	8	4	5	1	2
2	4	8	7	5	1	2	9	3	6
3	1	2	5	9	6	3	8	7	4
4	9	3	2	6	5	1	4	8	7
5	5	6	8	2	4	7	3	9	1
6	7	4	1	3	9	8	6	2	5
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1	6	9	3	7	8	4	5	1	2
2	4	8	7	5	1	2	9	3	6
3	1	2	5	9	6	3	8	7	4
4	9	3	2	6	5	1	4	8	7
5	5	6	8	2	4	7	3	9	1
6	7	4	1	3	9	8	6	2	5
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  - Unique elements x and y in Q such that: $a \cdot x = b$ and $y \cdot a = b$.

Quasigroup Completion Problems (QCPs)

							1	
4								
	2							
				5		4		7
		8						
		1		9				
3			4					
	5		1					
			8		6			

Definition (Quasigroup Completion Problem (QCP))

Given a partial quasigroup, identify a complete assignment such that each symbol occurs exactly once in each row and exactly once in each column, or prove that such an assignment does not exist.

QCPs as a SAT Problem

Variables

- n Boolean variables per cell,
- Each variable represents a number assigned to a cell,
- Variable q_{xyz} is assigned true iff the cell in row x, column y is assigned the number z.
- Total number of variables is n^3.

QCPs as a SAT Problem (Minimal Encoding)

Constraints

- At least one number must be assigned to each cell:
  $\bigwedge_{x=1}^{n} \bigwedge_{y=1}^{n} \bigvee_{z=1}^{n} q_{xyz}$
- No number is repeated in the same row:
  $\bigwedge_{y=1}^{n} \bigwedge_{z=1}^{n} \bigwedge_{x=1}^{n-1} \bigwedge_{i=x+1}^{n} (\neg q_{xyz} \vee \neg q_{xiz})$
- No number is repeated in the same column:
  $\bigwedge_{x=1}^{n} \bigwedge_{z=1}^{n} \bigwedge_{y=1}^{n-1} \bigwedge_{i=y+1}^{n} (\neg q_{xyz} \vee \neg q_{iyz})$

QCPs as a SAT Problem (Extended Encoding)

Constraints

- Each number must appear at least once in each row:
  $\bigwedge_{x=1}^{n} \bigwedge_{z=1}^{n} \bigvee_{y=1}^{n} q_{xyz}$
- Each number must appear at least once in each column:
  $\bigwedge_{y=1}^{n} \bigwedge_{z=1}^{n} \bigvee_{x=1}^{n} q_{xyz}$
- No two numbers are assigned to the same cell:
  $\bigwedge_{x=1}^{n} \bigwedge_{y=1}^{n} \bigwedge_{z=1}^{n-1} \bigwedge_{i=z+1}^{n} (\neg q_{xyz} \vee \neg q_{xyi})$

Symmetry Breaking in QCPs

6	9	3	7	8	4	5	1	2
4	8	7	5	1	2	9	3	6
1	2	5	9	6	3	8	7	4
9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
3	1	9	4	7	5	2	6	8
8	5	6	1	2	9	7	4	3
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4	8	7	5	1	2	9	3	6
1	2	5	9	6	3	8	7	4
9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
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Symmetry Breaking in QCPs

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9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
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7	4	1	3	9	8	6	2	5
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Symmetry Breaking in QCPs

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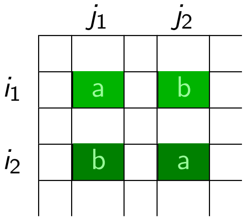
9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
3	1	9	4	2	5	7	6	8
8	5	6	1	7	9	2	4	3
2	7	4	8	3	6	1	5	9

- To break this symmetry we impose a lexicographical order between the two numbers.

Local Symmetry – *lsym*$_{22}$ (2 rows; 2 columns)

- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_2, j_1]$



Local Symmetry – $lsym_{22}$ (2 rows; 2 columns)

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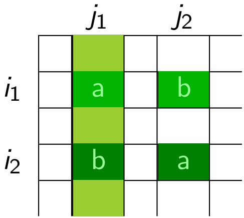

- Impose a lexicographical order on the subset of j_1:
  1. $\forall_{b>a}\ \neg(q_{i_1 j_1 b} \wedge q_{i_1 j_2 a} \wedge q_{i_2 j_1 a} \wedge q_{i_2 j_2 b})$
  2. $\forall_{a>b}\ \neg(q_{i_1 j_1 a} \wedge q_{i_1 j_2 b} \wedge q_{i_2 j_1 b} \wedge q_{i_2 j_2 a})$

Local Symmetry – $lsym_{22}$ (2 rows; 2 columns)

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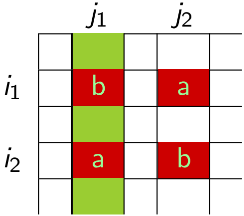

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  2. $\forall_{a>b}\ \neg(q_{i_1 j_1 a} \wedge q_{i_1 j_2 b} \wedge q_{i_2 j_1 b} \wedge q_{i_2 j_2 a})$

Local Symmetry – $lsym_{22}$ (2 rows; 2 columns)

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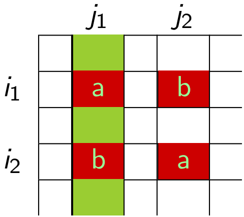

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  1. $\forall_{b>a}\ \neg(q_{i_1 j_1 b} \wedge q_{i_1 j_2 a} \wedge q_{i_2 j_1 a} \wedge q_{i_2 j_2 b})$
  2. $\forall_{a>b}\ \neg(q_{i_1 j_1 a} \wedge q_{i_1 j_2 b} \wedge q_{i_2 j_1 b} \wedge q_{i_2 j_2 a})$

6	9	3	7	8	4	5	1	2
4	8	7	5	1	2	3	9	6
1	2	5	9	6	3	8	7	4
9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	9	3	1
7	4	1	3	9	8	6	2	5
3	1	9	4	2	5	7	6	8
8	5	6	1	7	9	2	4	3
2	7	4	8	3	6	1	5	9

$$\neg(q_{i_2 j_7 9} \wedge q_{i_2 j_8 3} \wedge q_{i_5 j_7 3} \wedge q_{i_5 j_8 9})$$
$$\neg(q_{i_7 j_5 7} \wedge q_{i_7 j_6 2} \wedge q_{i_8 j_5 2} \wedge q_{i_8 j_6 7})$$

Local Symmetry – $lsym_{22}$ (Example)

6	9	3	7	8	4	5	1	2
4	8	7	5	1	2	9	3	6
1	2	5	9	6	3	8	7	4
9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
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$$\neg(q_{i_7 j_5 7} \wedge q_{i_7 j_6 2} \wedge q_{i_8 j_5 2} \wedge q_{i_8 j_6 7})$$

Local Symmetry – $lsym_{22}$ (Example)

6	9	3	7	8	4	5	1	2
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3	1	9	4	7	5	2	6	8
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$$\neg(q_{i_2 j_7 9} \wedge q_{i_2 j_8 3} \wedge q_{i_5 j_7 3} \wedge q_{i_5 j_8 9})$$

$$\neg(q_{i_7 j_5 7} \wedge q_{i_7 j_6 2} \wedge q_{i_8 j_5 2} \wedge q_{i_8 j_6 7})$$

Local Symmetry – $lsym_{23}$ (2 rows; 3 columns)

6	9	3	7	8	4	5	1	2
4	8	7	5	1	2	9	3	6
1	2	5	9	6	3	8	7	4
9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
3	1	9	4	7	5	2	6	8
8	5	6	1	2	9	7	4	3
2	7	4	8	3	6	1	5	9

Local Symmetry – *lsym*$_{23}$ (2 rows; 3 columns)

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4	8	7	5	1	2	9	3	6
1	2	5	9	6	3	8	7	4
9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
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Local Symmetry – *lsym*$_{23}$ (2 rows; 3 columns)

6	9	3	7	8	4	5	1	2
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9	4	2	3	5	1	6	8	7
5	6	8	2	4	7	3	9	1
7	3	1	6	9	8	4	2	5
3	1	9	4	7	5	2	6	8
8	5	6	1	2	9	7	4	3
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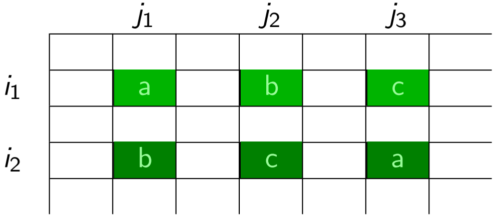
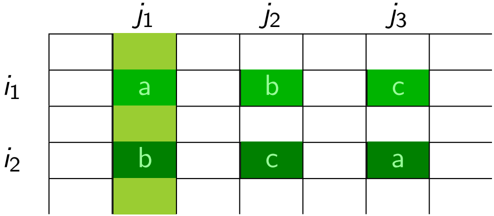
Local Symmetry – $lsym_{23}$ (1st Case)

- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_2, j_1]$



Local Symmetry – *lsym*$_{23}$ (1st Case)

- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_2, j_1]$



- Impose a lexicographical order on the subset of j_1:
  1. $\forall b > a : \neg(q_{i_1 j_1 b} \wedge q_{i_1 j_2 c} \wedge q_{i_1 j_3 a} \wedge q_{i_2 j_1 a} \wedge q_{i_2 j_2 b} \wedge q_{i_2 j_3 c})$
  2. $\forall a > b : \neg(q_{i_1 j_1 a} \wedge q_{i_1 j_2 b} \wedge q_{i_1 j_3 c} \wedge q_{i_2 j_1 b} \wedge q_{i_2 j_2 c} \wedge q_{i_2 j_3 a})$

- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_2, j_1]$

		j_1		j_2		j_3	
i_1		b		c		a	
i_2		a		b		c	

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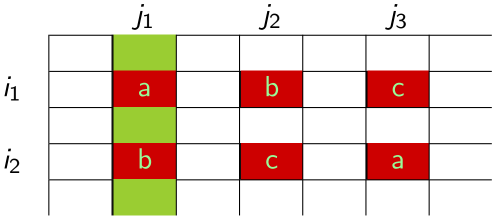
Local Symmetry – $lsym_{23}$ (1st Case)

- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_2, j_1]$



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  2. $\forall a > b : \neg(q_{i_1 j_1 a} \wedge q_{i_1 j_2 b} \wedge q_{i_1 j_3 c} \wedge q_{i_2 j_1 b} \wedge q_{i_2 j_2 c} \wedge q_{i_2 j_3 a})$

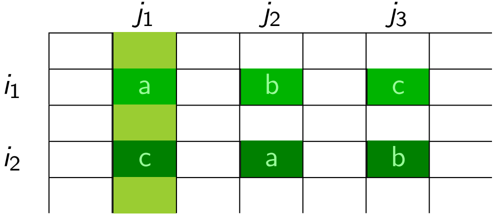
Local Symmetry – *lsym*$_{23}$ (2nd Case)

- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_2, j_1]$

		j_1		j_2		j_3	
i_1		a		b		c	
i_2		c		a		b	

Local Symmetry – $lsym_{23}$ (2nd Case)

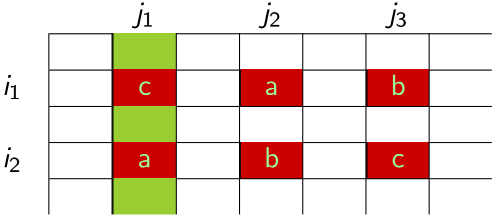
- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_2, j_1]$



- Impose a lexicographical order on the subset of j_1:
  1. $\forall c > a : \neg(q_{i_1 j_1 c} \wedge q_{i_1 j_2 a} \wedge q_{i_1 j_3 b} \wedge q_{i_2 j_1 a} \wedge q_{i_2 j_2 b} \wedge q_{i_2 j_3 c})$
  2. $\forall a > c : \neg(q_{i_1 j_1 a} \wedge q_{i_1 j_2 b} \wedge q_{i_1 j_3 c} \wedge q_{i_2 j_1 c} \wedge q_{i_2 j_2 a} \wedge q_{i_2 j_3 b})$

Local Symmetry – $lsym_{23}$ (2nd Case)

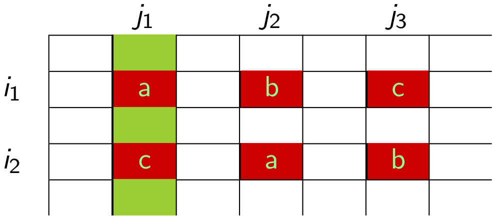
- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_2, j_1]$



- Impose a lexicographical order on the subset of j_1:
  1. $\forall c > a : \neg(q_{i_1 j_1 c} \land q_{i_1 j_2 a} \land q_{i_1 j_3 b} \land q_{i_2 j_1 a} \land q_{i_2 j_2 b} \land q_{i_2 j_3 c})$
  2. $\forall a > c : \neg(q_{i_1 j_1 a} \land q_{i_1 j_2 b} \land q_{i_1 j_3 c} \land q_{i_2 j_1 c} \land q_{i_2 j_2 a} \land q_{i_2 j_3 b})$

Local Symmetry – $lsym_{23}$ (2nd Case)

- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_2, j_1]$



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Local Symmetry – $lsym_{32}$ (3 rows; 2 columns)

6	9	3	7	8	4	5	1	2
4	8	7	5	1	2	9	3	6
1	2	5	9	6	3	8	7	4
9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
3	1	9	4	7	5	2	6	8
8	5	6	1	2	9	7	4	3
2	7	4	8	3	6	1	5	9

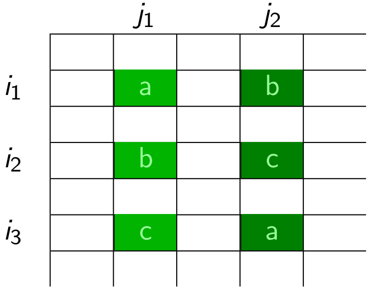
Local Symmetry – *lsym*$_{32}$ (3 rows; 2 columns)

6	9	3	7	8	4	5	1	2
4	8	7	5	1	2	9	3	6
1	2	5	9	6	3	8	7	4
9	3	2	6	5	1	4	8	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
3	1	9	4	7	5	2	6	8
8	5	6	1	2	9	7	4	3
2	7	4	8	3	6	1	5	9

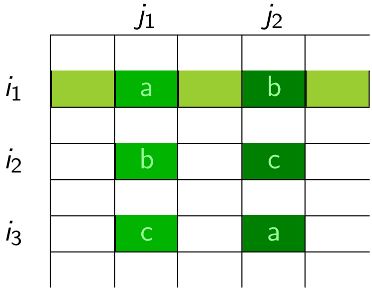
Local Symmetry – $lsym_{32}$ (3 rows; 2 columns)

6	9	3	7	8	4	5	1	2
4	8	7	5	1	2	9	3	6
1	2	5	9	6	3	7	8	4
9	3	2	6	5	1	8	4	7
5	6	8	2	4	7	3	9	1
7	4	1	3	9	8	6	2	5
3	1	9	4	7	5	2	6	8
8	5	6	1	2	9	4	7	3
2	7	4	8	3	6	1	5	9

- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_1, j_2]$



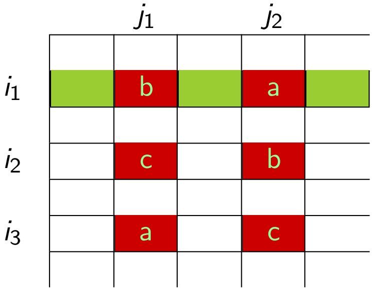
- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_1, j_2]$



- Impose a lexicographical order on the subset of i_1:
  1. $\forall b > a : \neg(q_{i_1 j_1 b} \wedge q_{i_1 j_2 a} \wedge q_{i_2 j_1 c} \wedge q_{i_2 j_2 b} \wedge q_{i_3 j_1 a} \wedge q_{i_3 j_2 c})$
  2. $\forall a > b : \neg(q_{i_1 j_1 a} \wedge q_{i_1 j_2 b} \wedge q_{i_2 j_1 b} \wedge q_{i_2 j_1 c} \wedge q_{i_3 j_1 c} \wedge q_{i_3 j_2 a})$

Local Symmetry – $lsym_{32}$ (1st Case)

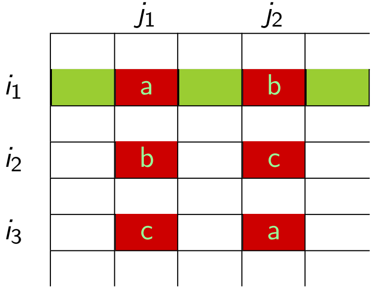
- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_1, j_2]$



- Impose a lexicographical order on the subset of i_1:
  1. $\forall b > a : \neg(q_{i_1 j_1 b} \wedge q_{i_1 j_2 a} \wedge q_{i_2 j_1 c} \wedge q_{i_2 j_2 b} \wedge q_{i_3 j_1 a} \wedge q_{i_3 j_2 c})$
  2. $\forall a > b : \neg(q_{i_1 j_1 a} \wedge q_{i_1 j_2 b} \wedge q_{i_2 j_1 b} \wedge q_{i_2 j_1 c} \wedge q_{i_3 j_1 c} \wedge q_{i_3 j_2 a})$

Local Symmetry – $lsym_{32}$ (1st Case)

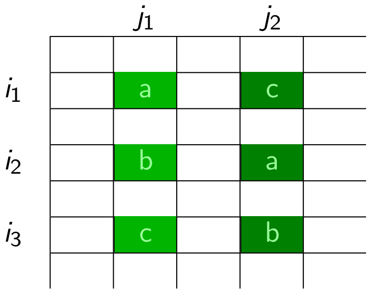
- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_1, j_2]$



- Impose a lexicographical order on the subset of i_1:
  1. $\forall b > a : \neg(q_{i_1 j_1 b} \wedge q_{i_1 j_2 a} \wedge q_{i_2 j_1 c} \wedge q_{i_2 j_2 b} \wedge q_{i_3 j_1 a} \wedge q_{i_3 j_2 c})$
  2. $\forall a > b : \neg(q_{i_1 j_1 a} \wedge q_{i_1 j_2 b} \wedge q_{i_2 j_1 b} \wedge q_{i_2 j_1 c} \wedge q_{i_3 j_1 c} \wedge q_{i_3 j_2 a})$

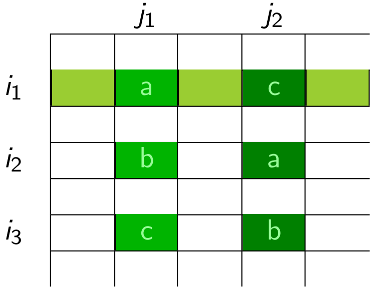
Local Symmetry – $lsym_{32}$ (2nd Case)

- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_1, j_2]$



Local Symmetry – $lsym_{32}$ (2nd Case)

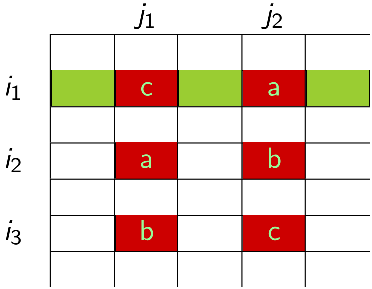
- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_1, j_2]$



- Impose a lexicographical order on the subset of i_1:
  1. $\forall c > a : \neg(q_{i_1 j_1 c} \wedge q_{i_1 j_2 a} \wedge q_{i_2 j_1 a} \wedge q_{i_2 j_2 b} \wedge q_{i_3 j_1 b} \wedge q_{i_3 j_2 c})$
  2. $\forall a > c : \neg(q_{i_1 j_1 a} \wedge q_{i_1 j_2 c} \wedge q_{i_2 j_1 b} \wedge q_{i_2 j_1 a} \wedge q_{i_3 j_1 c} \wedge q_{i_3 j_2 b})$

Local Symmetry – $lsym_{32}$ (2nd Case)

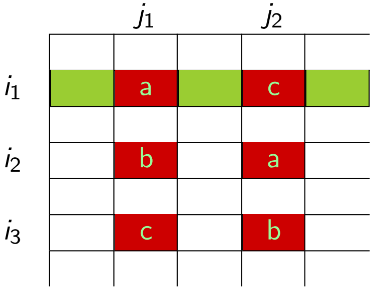
- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_1, j_2]$



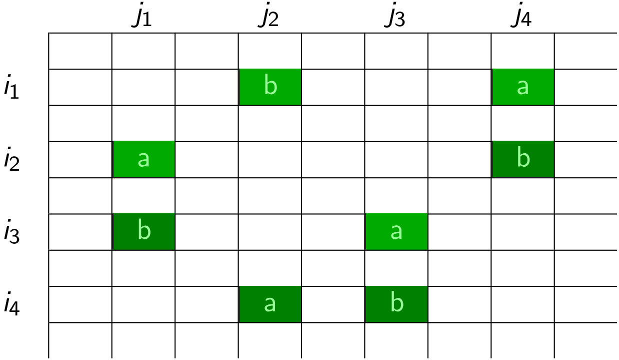
- Impose a lexicographical order on the subset of i_1:
  1. $\forall c > a : \neg(q_{i_1 j_1 c} \wedge q_{i_1 j_2 a} \wedge q_{i_2 j_1 a} \wedge q_{i_2 j_2 b} \wedge q_{i_3 j_1 b} \wedge q_{i_3 j_2 c})$
  2. $\forall a > c : \neg(q_{i_1 j_1 a} \wedge q_{i_1 j_2 c} \wedge q_{i_2 j_1 b} \wedge q_{i_2 j_1 a} \wedge q_{i_3 j_1 c} \wedge q_{i_3 j_2 b})$

Local Symmetry – *$lsym_{32}$* (2nd Case)

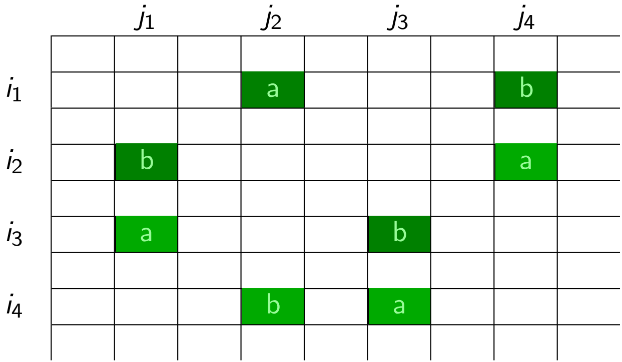
- $\mathcal{Q}[i_1, j_1] < \mathcal{Q}[i_1, j_2]$



- Impose a lexicographical order on the subset of i_1:
  1. $\forall c > a : \neg(q_{i_1 j_1 c} \land q_{i_1 j_2 a} \land q_{i_2 j_1 a} \land q_{i_2 j_2 b} \land q_{i_3 j_1 b} \land q_{i_3 j_2 c})$
  2. $\forall a > c : \neg(q_{i_1 j_1 a} \land q_{i_1 j_2 c} \land q_{i_2 j_1 b} \land q_{i_2 j_1 a} \land q_{i_3 j_1 c} \land q_{i_3 j_2 b})$


j_1
j_2
j_3
j_4
i_1
b
a
i_2
a
b
i_3
b
a
i_4
a
b

Other Examples of Local Symmetries



Experimental Results (Satisfiable Instances)

- Reduction (in percentage) of the number of solutions when using the different encodings

$lsym_{22}$	$lsym_{23}$	$lsym_{32}$	$lsym_{all}$
77.191	8.910	10.934	81.668

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$lsym_{22}$	$lsym_{23}$	$lsym_{32}$	$lsym_{all}$
77.191	8.910	10.934	81.668

- Satisfiable instances using `satz` with a time limit of 6000s

Order	w/o *lsym*		$lsym_{22}$		$lsym_{32}$		$lsym_{23}$	
35	100	0.66	100	0.64	100	0.825	100	0.635
37	100	3.44	100	3.37	100	3.495	100	4.015
40	100	18.76	100	18.63	100	26.645	100	19.92
43	90	120.66	91	134.41	90	156.11	89	170.655
45	68	665.22	69	633.55	68	802.835	70	740.22

Experimental Results (blindsatz)

- `satz`

  The look-ahead heuristic implemented in `satz` chooses the variable that once assigned will imply the highest number of assignments due to unit propagation.

Experimental Results (blindsatz)

- satz

  The look-ahead heuristic implemented in satz chooses the variable that once assigned will imply the highest number of assignments due to unit propagation.

- blindsatz

  We simply choose the *first unassigned variable* to branch on. This makes the heuristic to choose the variables following a fixed order which is a non-biased approach.

Experimental Results (Satisfiable Instances)

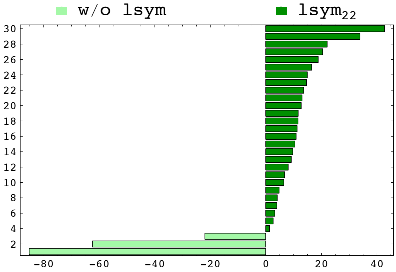
- Satisfiable instances using blind`satz` with a time limit of 1000s

w/o *lsym*	$lsym_{22}$	$lsym_{32}$	$lsym_{23}$	$lsym_{all}$
88.97	81.57	88.87	88.97	81.095

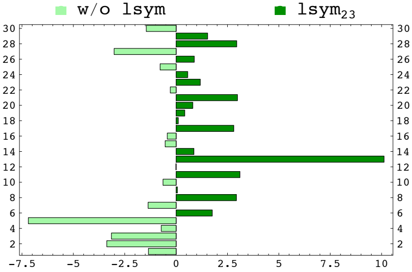
Experimental Results (Satisfiable Instances)

- Satisfiable instances using blindsatz with a time limit of 1000s

w/o lsym	$lsym_{22}$	$lsym_{32}$	$lsym_{23}$	$lsym_{all}$
88.97	81.57	88.87	88.97	81.095



27 Instances (1.32;12.57;42.74)%

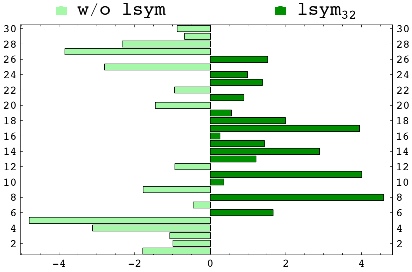


16 Instances (0.06;2.07;10.12)%

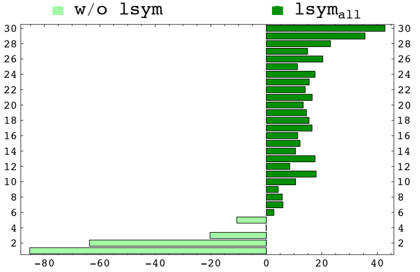
Experimental Results (Satisfiable Instances)

- Satisfiable instances using blindsatz with a time limit of 1000s

w/o lsym	$lsym_{22}$	$lsym_{32}$	$lsym_{23}$	$lsym_{all}$
88.97	81.57	88.87	88.97	81.095



15 Instances (0.26;1.85;4.59)%



25 Instances (2.76;15.17;42.81)%

Experimental Results (Unsatisfiable Instances)

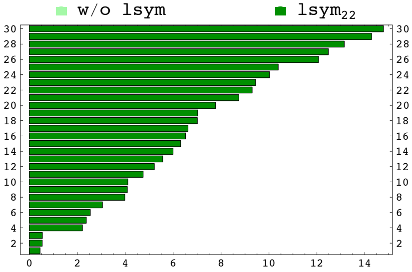
- Unsatisfiable instances using blind`satz` with a time limit of 1000s

w/o lsym	$lsym_{22}$	$lsym_{32}$	$lsym_{23}$	$lsym_{all}$
376.075	360.655	378.52	377.955	358.47

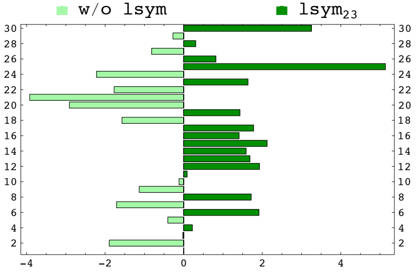
Experimental Results (Unsatisfiable Instances)

- Unsatisfiable instances using blindsatz with a time limit of 1000s

w/o lsym	$lsym_{22}$	$lsym_{32}$	$lsym_{23}$	$lsym_{all}$
376.075	360.655	378.52	377.955	358.47



30 Instances (0.45;6.71;14.78)%

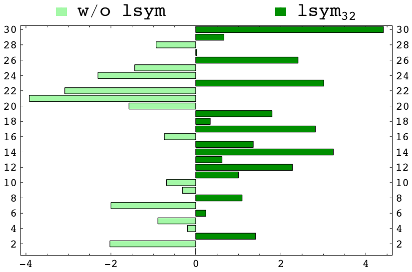


16 Instances (0.09;1.69;5.14)%

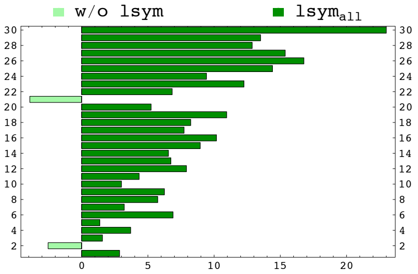
Experimental Results (Unsatisfiable Instances)

- Unsatisfiable instances using blindsatz with a time limit of 1000s

w/o lsym	$lsym_{22}$	$lsym_{32}$	$lsym_{23}$	$lsym_{all}$
376.075	360.655	378.52	377.955	358.47



16 Instances (0.02;1.67;4.41)%



28 Instances (1.37;8.42;23)%

Conclusions & Future Work

- Identified several types of Local Symmetries in QCPs
- Added new clauses for symmetry breaking has a negative impact on the solver

- Simplify the clauses needed to break Local Symmetries
- Develop new heuristics to cope with Local Symmetries
- Apply a similar reasoning to other problems