Preprocessing in Pseudo-Boolean Optimization: An Experimental Evaluation

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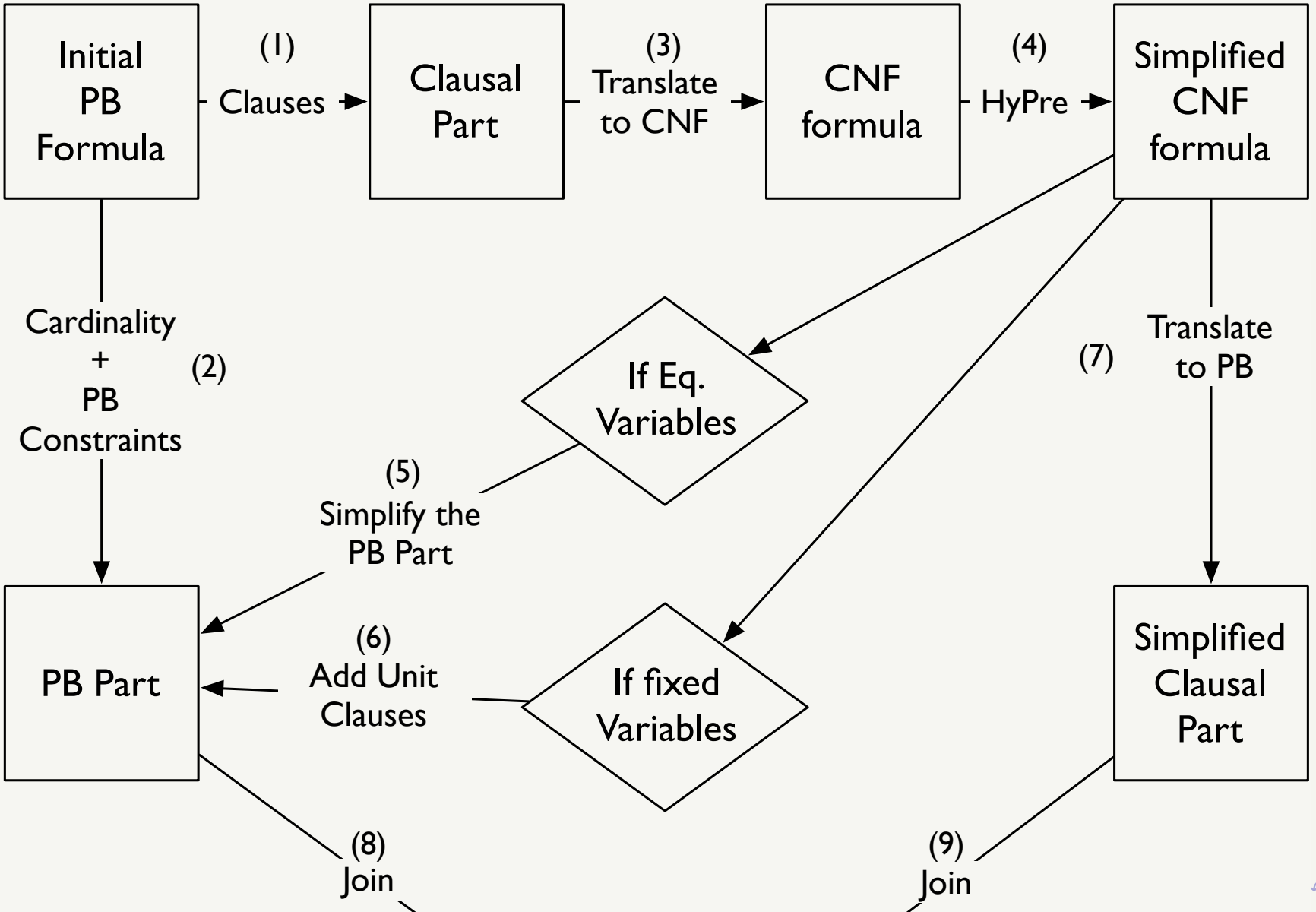
Motivation

- Preprocessing is a powerful tool in SAT solving:
  - e.g. `HyPre`, `NiVER`, `SatELite`, `ReVivAl`, `extHyPre`, ...
- PBO is closely related to SAT
- However, preprocessing is almost non existent in PBO
- We propose to benefit from the advances in SAT preprocessing and use them in PBO
- In this talk, we will show how we can use `HyPre` and `extHyPre` to preprocess a PBO instance

HyPre in PBO


Initial PB Formula
(1) Clauses
Clausal Part
(3) Translate to CNF
CNF formula
(4) HyPre
Simplified CNF formula
Cardinality + PB Constraints
(2)
If Eq. Variables
(7) Translate to PB
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If fixed Variables
Simplified Clausal Part
(8) Join
(9) Join

HyPre in PBO

$x_1 \vee x_2 \vee x_3$

$\sim x_1 \vee x_4$

$\sim x_2 \vee x_4$

$\sim x_3 \vee x_4$

HyPre in PBO

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$x_1 \vee x_2 \vee x_3$

$x_1 \vee x_2 \vee x_3$

$x_1 \vee x_2 \vee x_3$

$\sim x_3 \vee x_4$

Background (SAT and PBO)

- SAT (Boolean Satisfiability)
  - CNF formula $\varphi = \bigvee x_j$
  - *SAT problem* is to dedice if $\bigwedge_i \varphi_i$ is satisfiable
- PBO (Pseudo-Boolean Optimization)
  - PB constraints $\varphi = \sum_j a_j x_j \geq b$
  - Linear objective function $f = \sum_j c_j x_j$
  - *PBO problem* is to find an assignment to x_j such that all φ_i are satisfied and the value of f is optimized

Background (PB Constraints)

- PB constraints can be divided into three categories:
  - *cardinality constraints*:

    e.g. $x_1 + x_2 + x_3 + \overline{x}_4 \geq 2$
  - *clauses*:

    e.g. $x_1 + \overline{x}_2 + x_3 \geq 1$
  - *general PB constraint:*

    e.g. $7x_1 + 4\overline{x}_2 + 5x_3 \geq 7$
- This categories form two distinct parts:
  - The *PB part* is formed by the conjunction of cardinality constraints and general PB constraints
  - The *clausal part* is formed by the conjunction of clauses

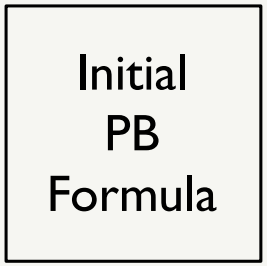
HyPre Preprocessor

- HyPre is based on *Hyper-Binary Resolution* (HypBinRes):

$$\frac{(l_1 \vee \ldots \vee l_n) \quad (\overline{l_1} \vee y) \ldots (\overline{l_{n-1}} \vee y)}{(y \vee l_n)} \quad \text{for } n \geq 2$$

- In addition to HypBinRes, HyPre also simplifies the formula through Equality Reduction and Unit Propagation.

HyPre in PBO



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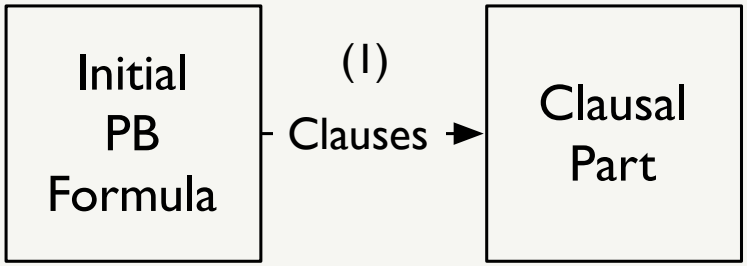

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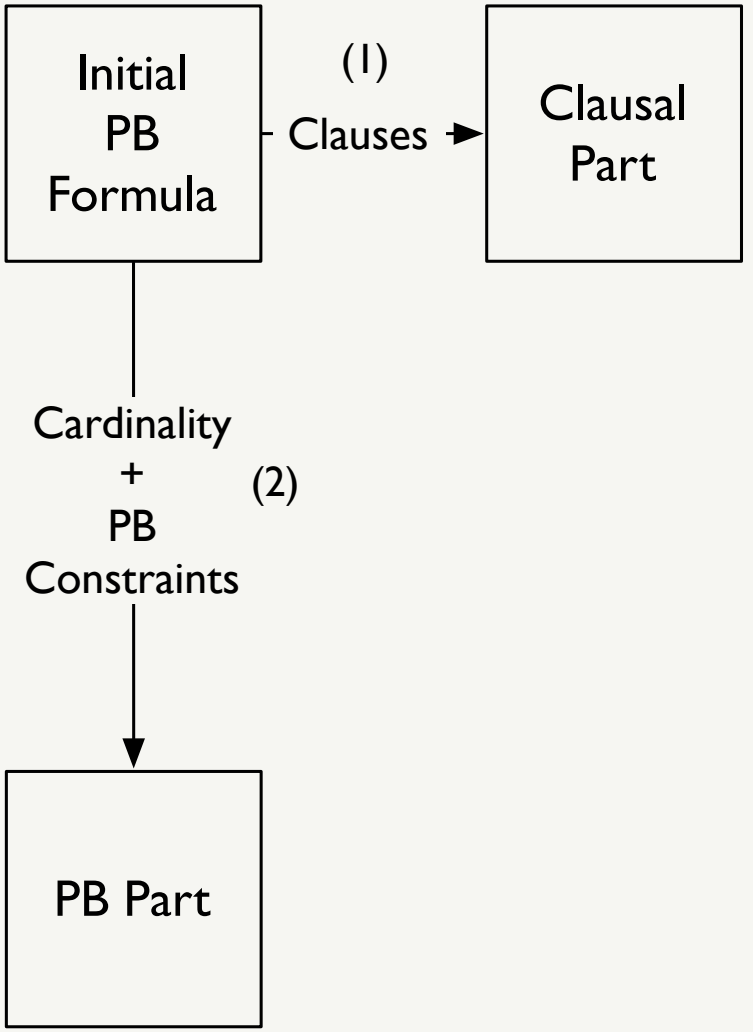
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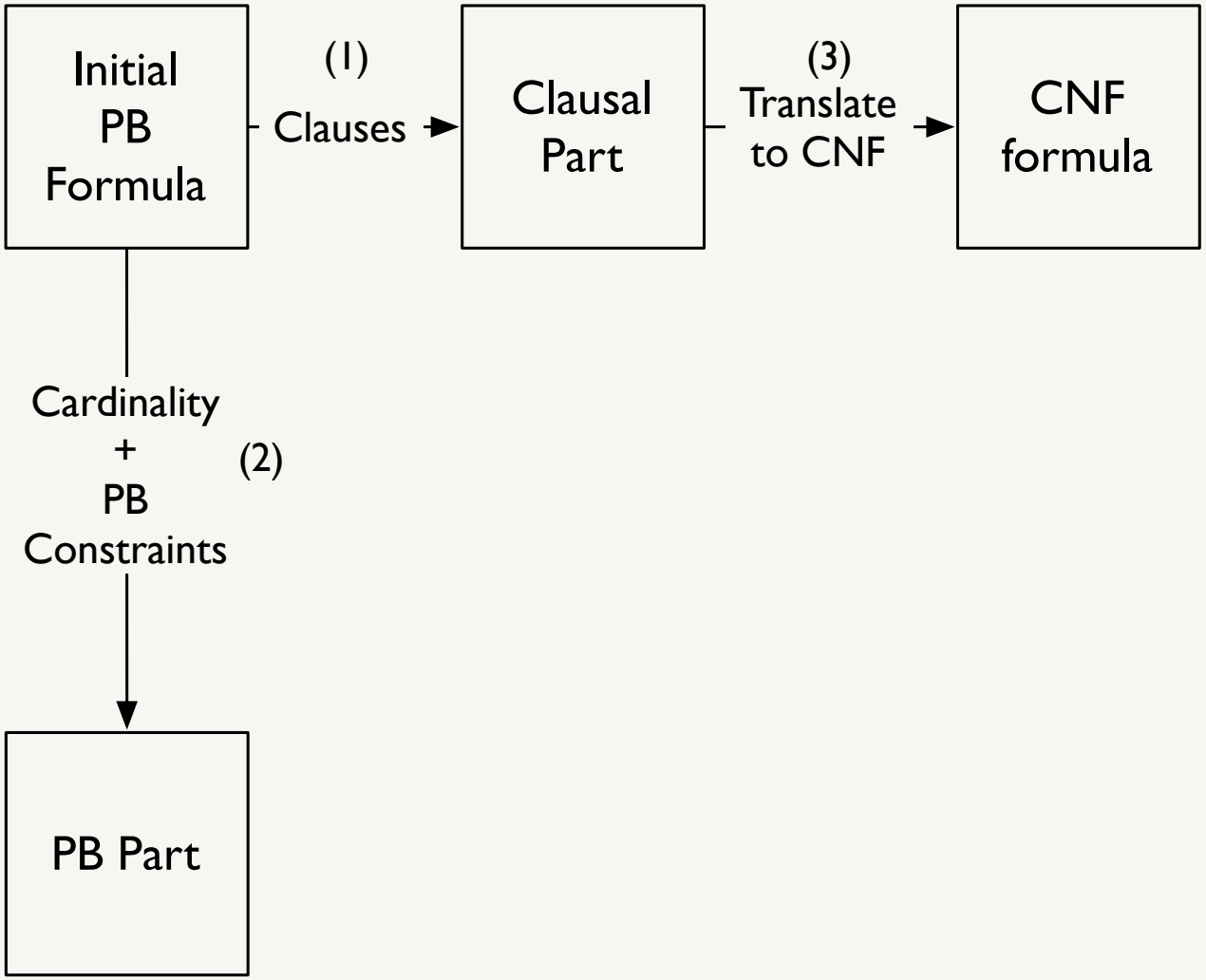
Formula
(1)
Clauses
Clausal
Part
Cardinality
+
PB
Constraints
(2)
PB Part

HyPre in PBO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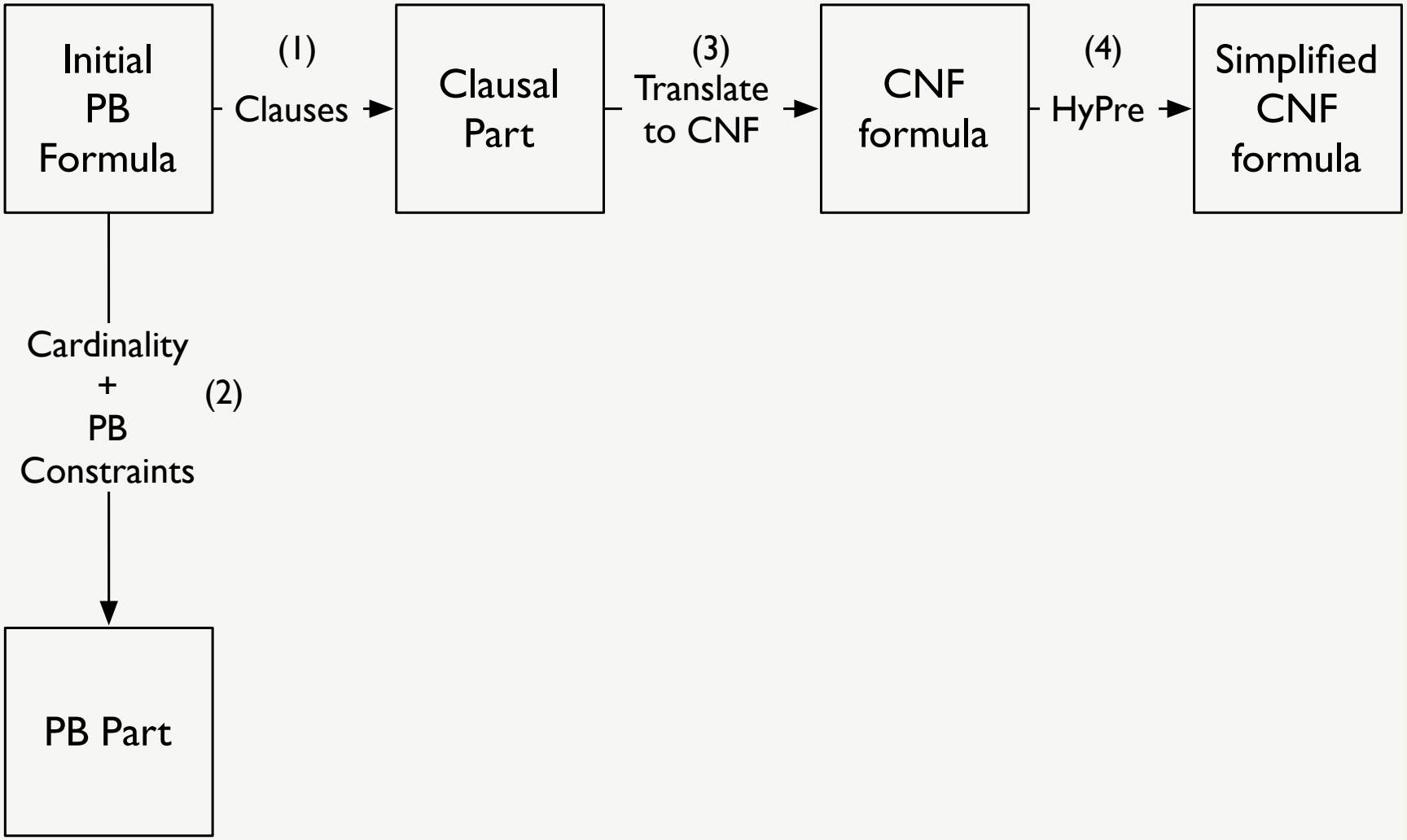
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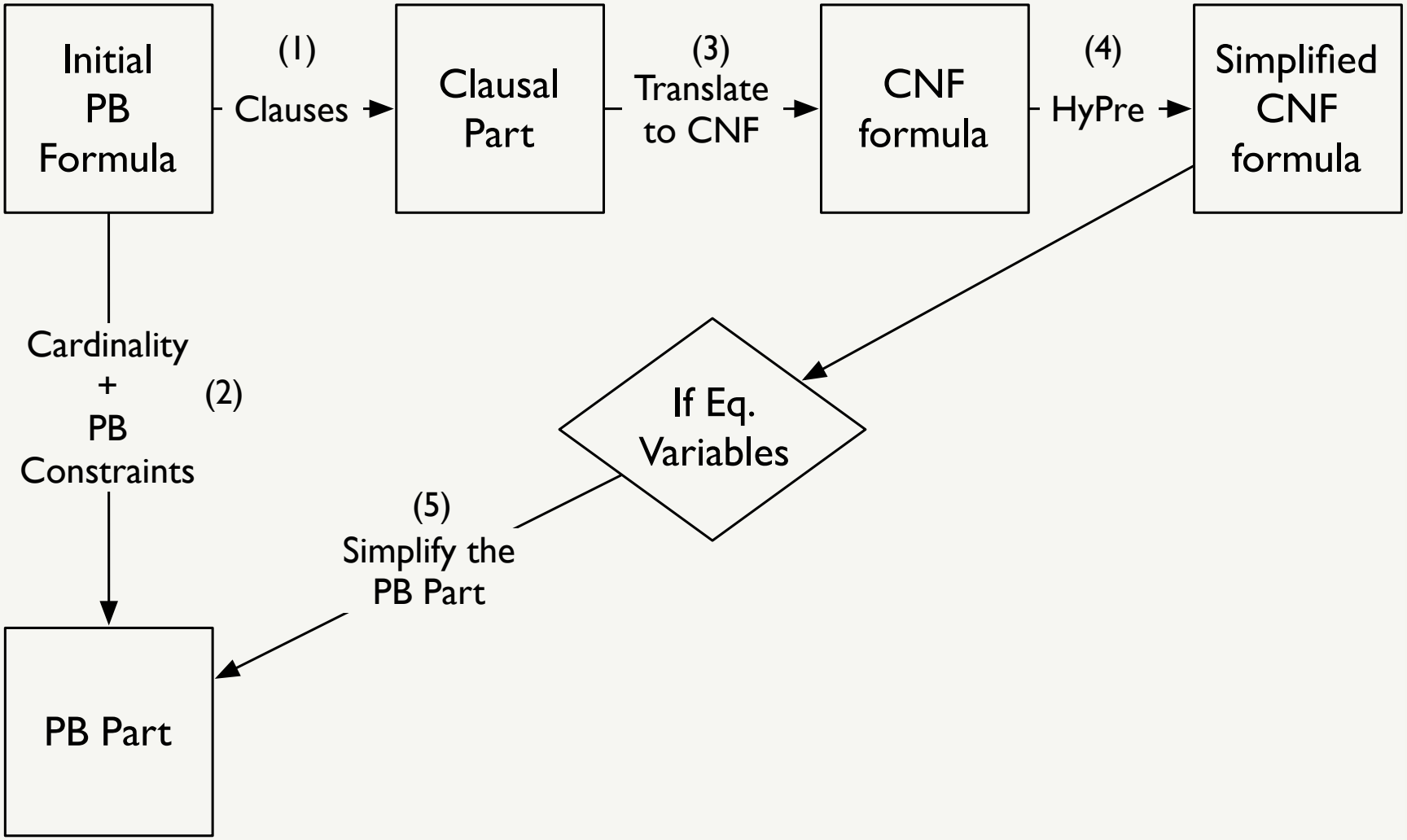
HyPre in PBO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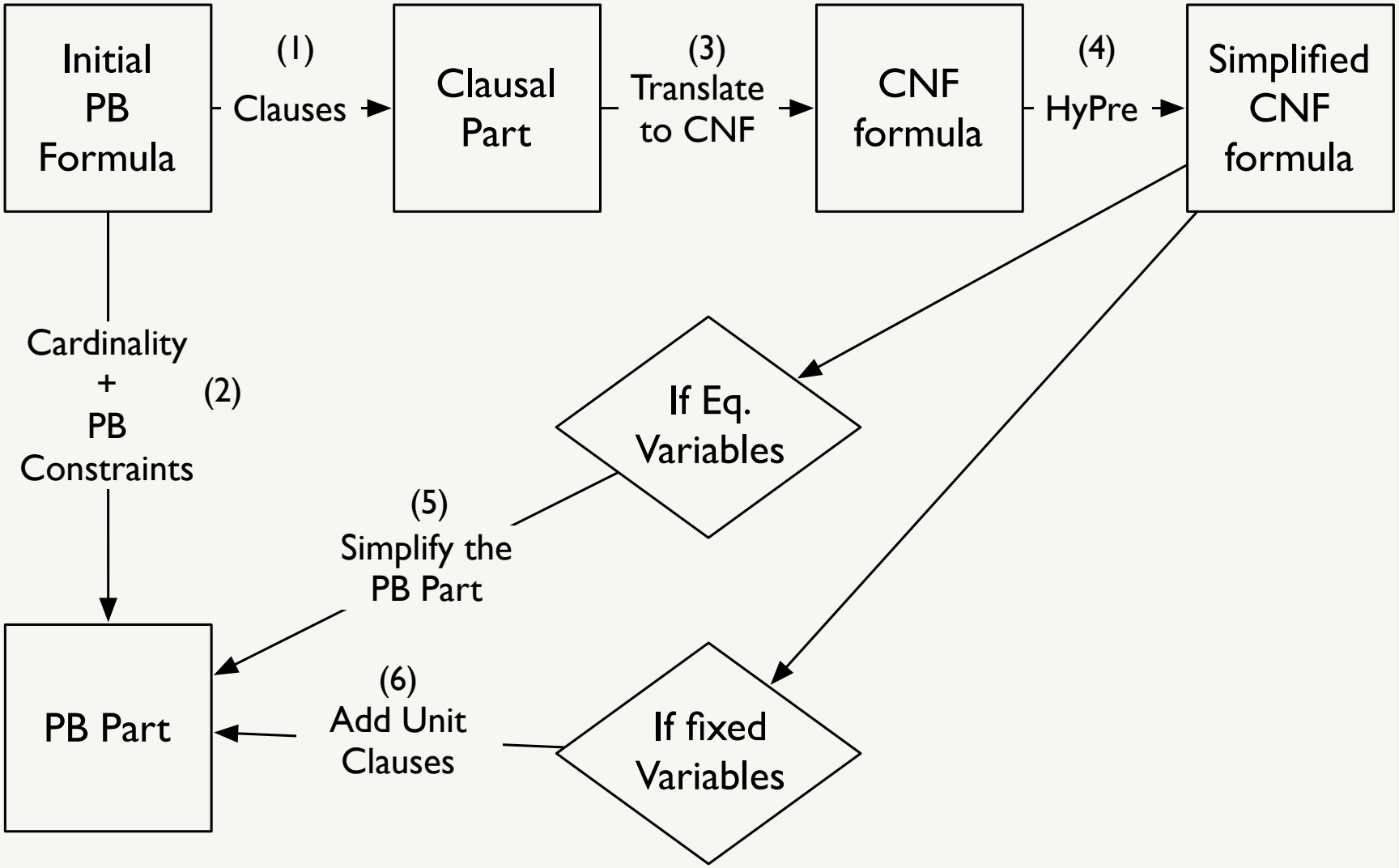
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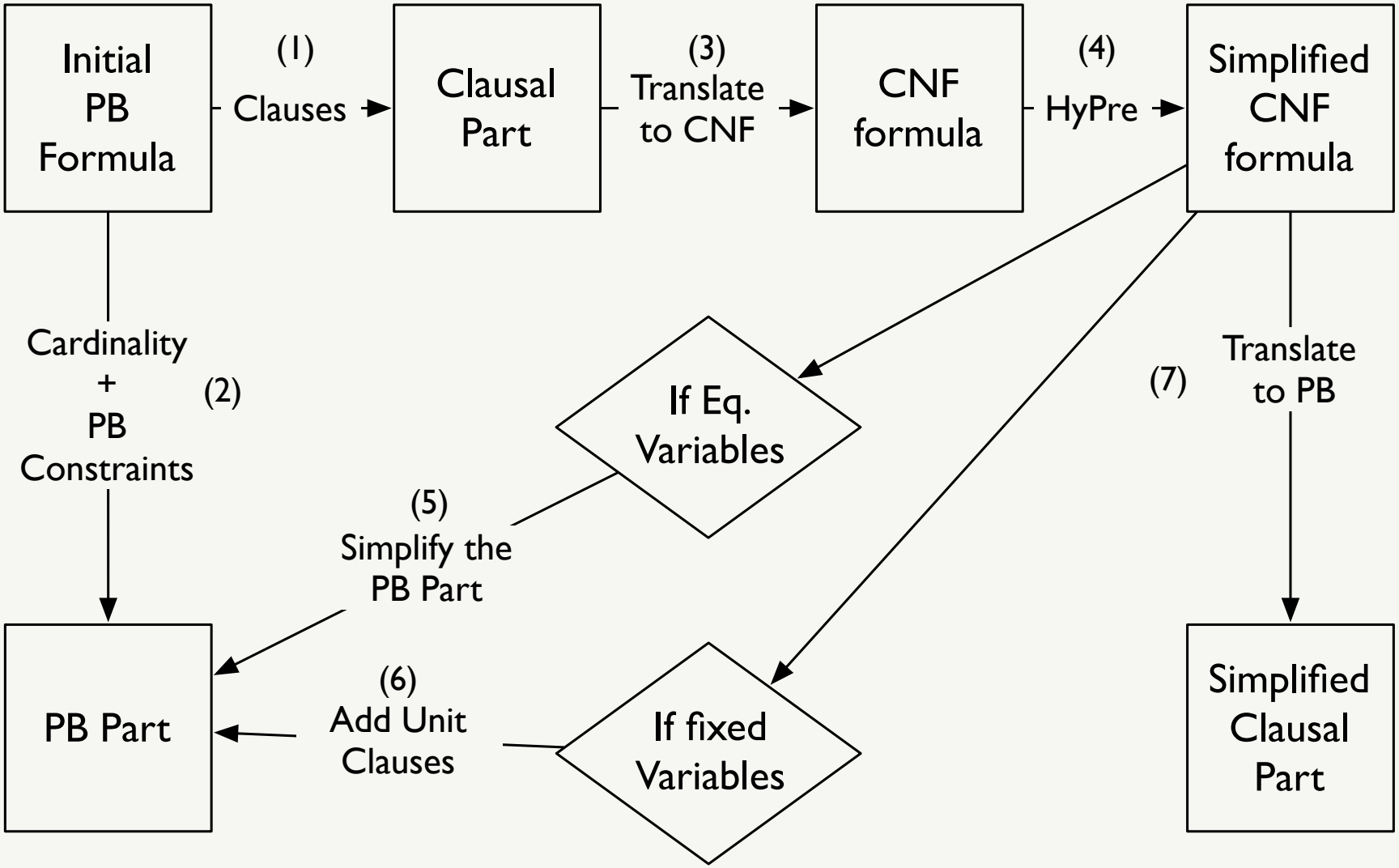
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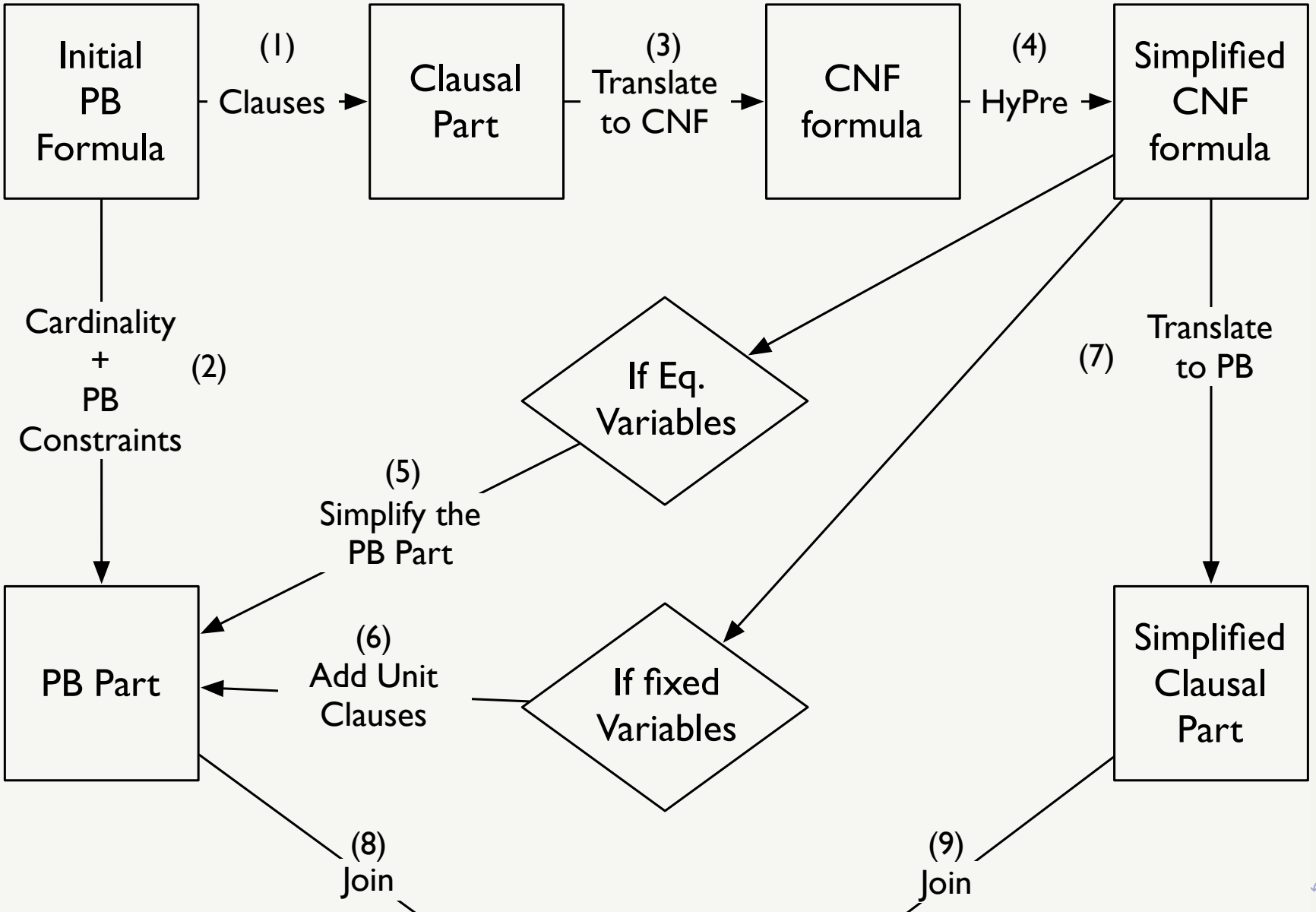
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`extHyPre` Preprocessor

- `extHyPre` is based on *Extended Hyper Resolution (ExtHypRes)*, which is an extension of *HypBinRes*:

$$\frac{(l_1 \vee l_2 \vee \ldots \vee l_n) \quad (\overline{l_1} \vee \alpha_1), \ldots, (\overline{l_n} \vee \alpha_n)}{(\bigcup_{1 \leq i \leq n}(\alpha_i))}$$

  where α_i for $1 \leq i \leq n$ are sub-clauses.

- `extHyPre` produces new resolvent clauses that are added to the original formula

New graph representation of a CNF formula

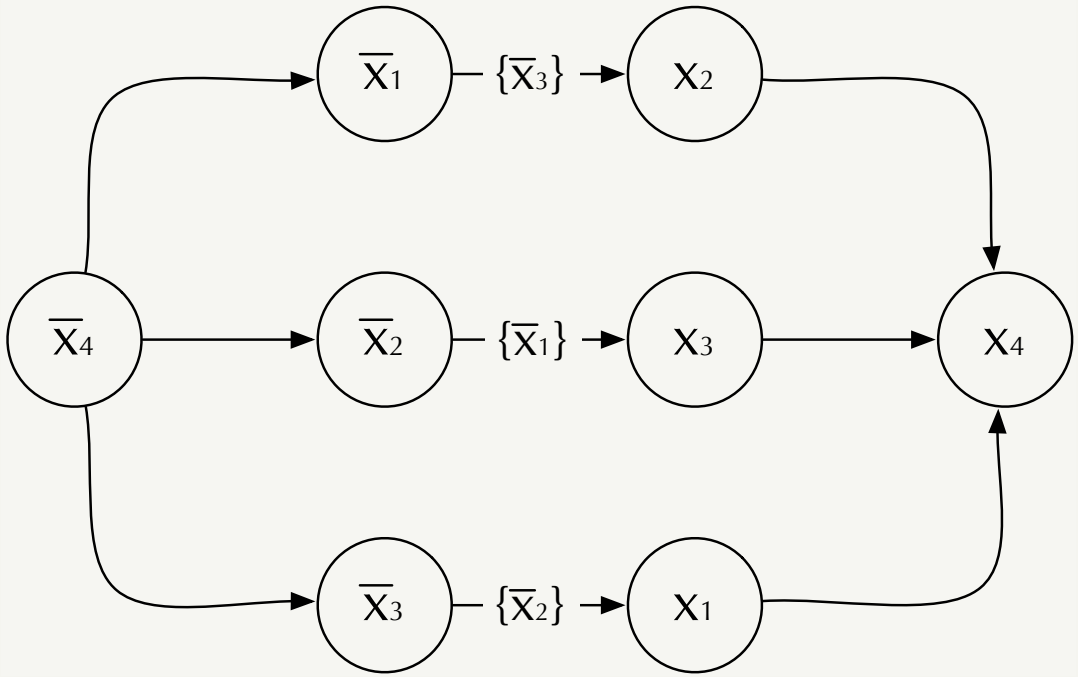
- `extHyPre` is applied through a new graph representation of a CNF formula:
  - The idea behind this representation is to extend the binary implication graph for clauses of any size

$x_1 \vee x_2 \vee x_3$

$\overline{x}_1 \vee x_4$

$\overline{x}_2 \vee x_4$

$\overline{x}_3 \vee x_4$



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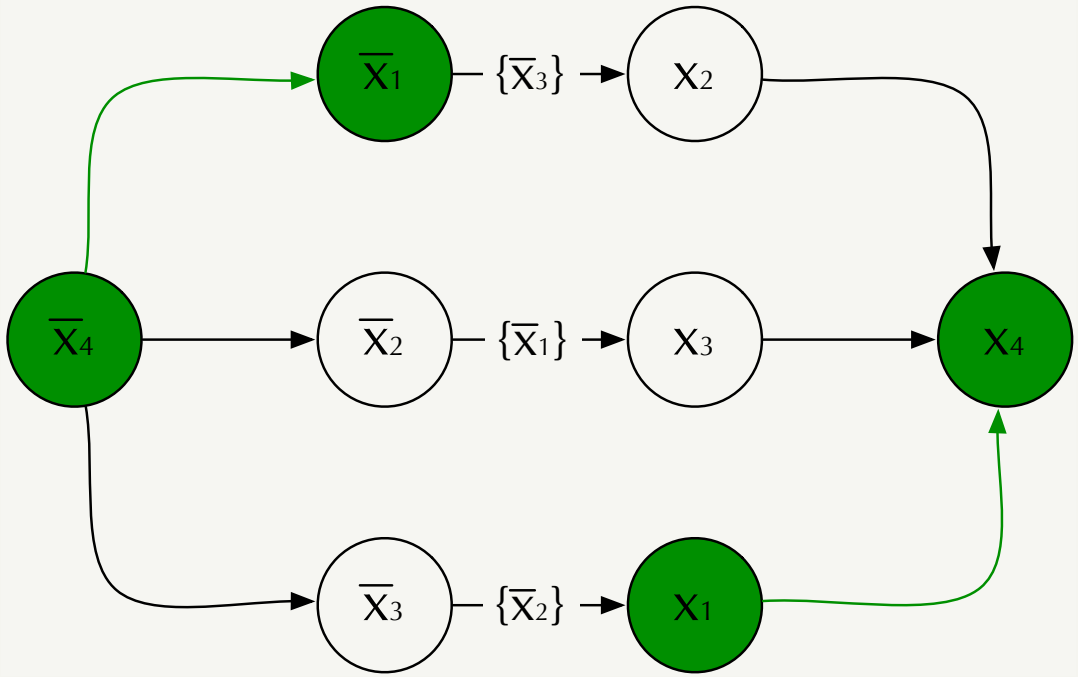

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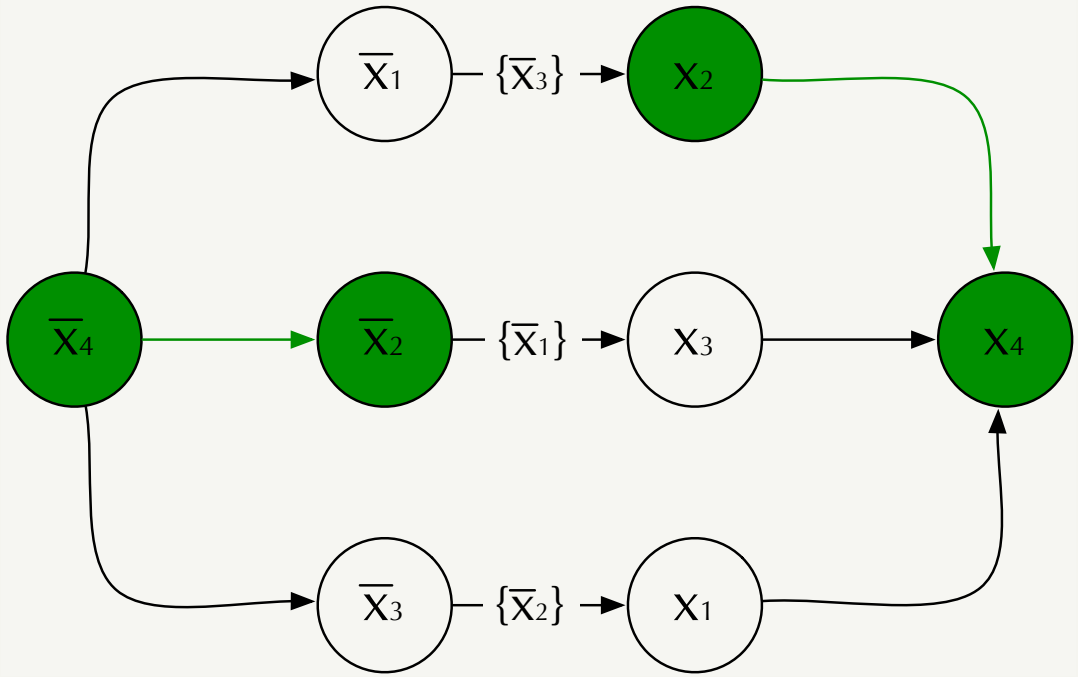

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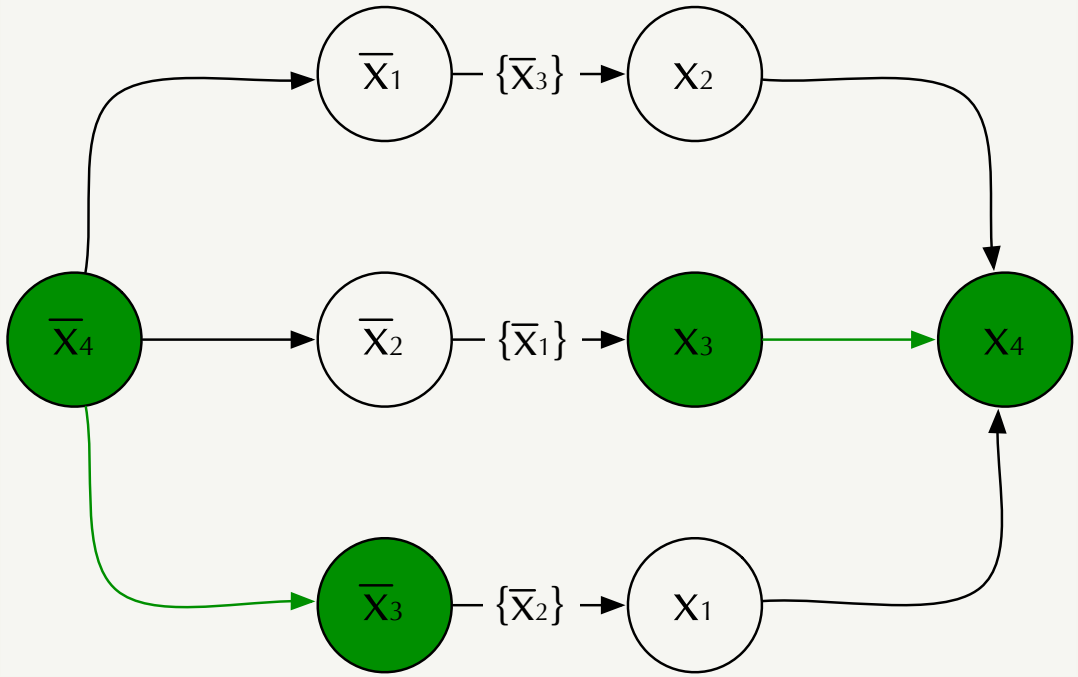

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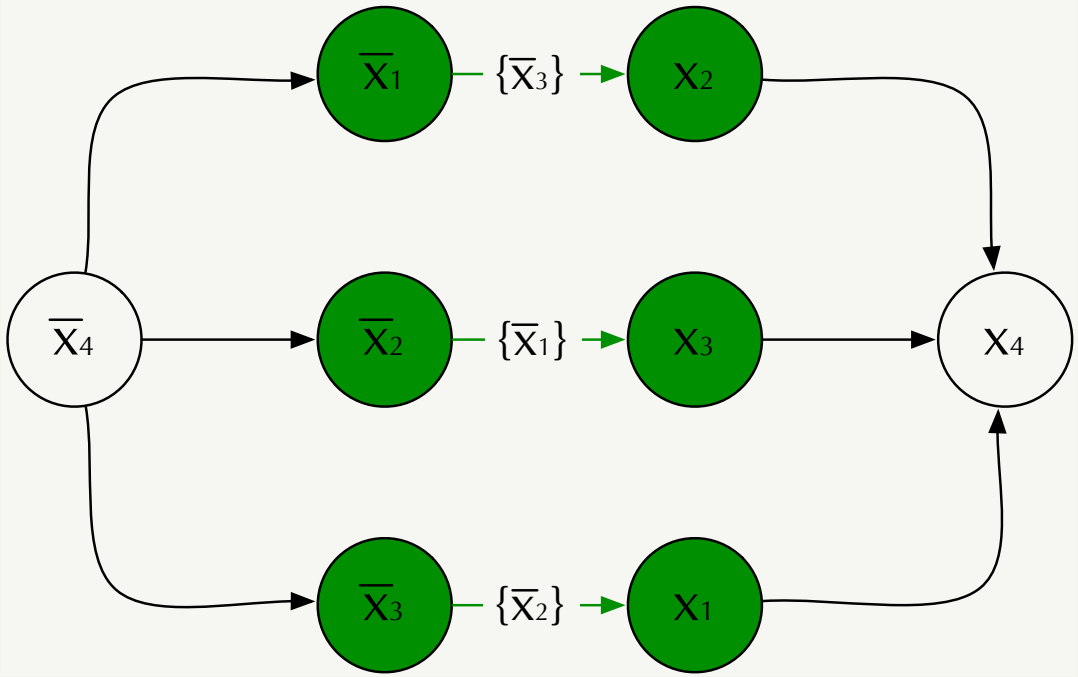

extHyPre in PBO

Enrich the graph representation:

- Information from *cardinality constraints*:
  - Given $x_1, \ldots, x_n \geq b$
  - Suppose that $n - b + 1$ literals are falsified
  - Then if one of the $b + 1$ literals is falsified it implies that all the remaining b literals are satisfied

- $x_1 + x_2 + x_3 + x_4 \geq 2$

$$\overline{x_1} \Rightarrow_{\{\overline{x_2}\}} x_3 \wedge \overline{x_1} \Rightarrow_{\{\overline{x_2}\}} x_4$$
$$\overline{x_2} \Rightarrow_{\{\overline{x_3}\}} x_4 \wedge \overline{x_2} \Rightarrow_{\{\overline{x_3}\}} x_1$$
$$\overline{x_3} \Rightarrow_{\{\overline{x_4}\}} x_1 \wedge \overline{x_3} \Rightarrow_{\{\overline{x_4}\}} x_2$$
$$\overline{x_4} \Rightarrow_{\{\overline{x_1}\}} x_2 \wedge \overline{x_4} \Rightarrow_{\{\overline{x_1}\}} x_3$$

extHyPre in PBO

Enrich the graph representation:

- Information from *general PB constraints*:
  - Probing on each literal of each *general PB constraint*
  - $2x_1 + x_2 + x_3 \geq 2$

    $\overline{x_1} \Rightarrow x_2$
    $\overline{x_1} \Rightarrow x_3$

Clause Selection Heuristics

- `extHyPre` uses two fixed parameters:
  - maximum size of the resolvent : 75
  - percentage of clauses to be considered: 10%
- Heuristics for clause selection:
  - random - `extHyPre(75,10,r)`
  - larger clauses - `extHyPre(75,10,c)`
  - clauses with higher weight - `extHyPre(75,10,d)`

Experimental Results (HyPre I)

- Average size of the 309 instances to be preprocessed by HyPre and average time of preprocessing

Benchmark	#	Vars	Constraints			Time
			Cls	Card	PB	
aksoy	79	15,259.90	53,156.57	0.13	0.00	0.21
logic-synthesis	74	1,767.91	1,605.35	0.00	0.00	0.01
primes-dimacs-cnf	130	1,688.45	11,537.59	0.00	0.00	0.04
routing	10	679.20	2,023.00	24.00	0.00	0.01
synthesis-ptl-cmos	8	375.75	879.26	0.00	0.00	0.01
haplotype	8	19,926.75	1,186,541.00	3,260.75	0.00	1.30

Experimental Results (HyPre II)

- Impact of HyPre preprocessing on bsolo, Pueblo and minisat+

Benchmark	#	bsolo		Pueblo		minisat+	
		w/o pre	HyPre	w/o pre	HyPre	w/o pre	HyPre
aksoy	79	26	*24*	15	15	24	24
logic-synthesis	74	51	*50*	31	**32**	31	31
primes-dimacs-cnf	130	69	69	75	**79**	79	**83**
routing	10	9	*8*	10	*9*	10	10
synthesis-ptl-cmos	8	6	6	1	1	1	1
haplotype	8	0	0	0	0	8	8
Total	309	161	*157*	132	**136**	153	**157**

Experimental Results (HyPre III)

- Details of some instances of HyPre preprocessing on bsolo

Benchmark	Instance	bsolo	
		w/o pre	HyPre
aksoy	f20c10b_016_area_delay	**25** ✔	25
	fir07_area_delay	**16** ✔	17
	fir09_area_opers	**34** ✔	43
	fir08_trarea_ac	**1769** ✔	2235
	f20c10b_029_area_delay	27	**24** ✔
	fir07_area_opers	34	**32** ✔
logic-synthesis	prom2.pi	**287** ✔	302
routing	s4-4-3-2pb	**64** ✔	64

Experimental Results (HyPre III)

- Details of some instances of HyPre preprocessing on Pueblo

Benchmark	Instance	Pueblo	
		w/o pre	HyPre
logic-synthesis	sao2.b	25	**25** ✔
primes-dimacs-cnf	ssa7552-038	1448	**1448** ✔
	ssa7552-158	1327	**1327** ✔
	ssa7552-159	1327	**1327** ✔
	ssa7552-160	1359	**1359** ✔
routing	s4-4-3-7pb	**64** ✔	64

Experimental Results (HyPre III)

- Details of some instances of HyPre preprocessing on minisat+

Benchmark	Instance	minisat+	
		w/o pre	HyPre
primes-dimacs-cnf	ssa7552-038	1448	**1448** ✔
	ssa7552-158	1327	**1327** ✔
	ssa7552-159	1327	**1327** ✔
	ssa7552-160	1359	**1359** ✔

Experimental Results (extHyPre I)

- Average time in seconds of extHyPre preprocessing using the different clause selection heuristics

Benchmark	#	extHyPre (75,10,c)	extHyPre (75,10,d)	extHyPre (75,10,r)
aksoy	79	78.51	95.83	80.61
logic-synthesis	74	3.48	3.51	3.43
primes-dimacs-cnf	130	0.34	0.39	0.62
radar	12	0.12	0.18	0.12
routing	10	0.13	0.13	0.11
synthesis-ptl-cmos	8	0.04	0.05	0.04
testset	6	0.01	0.01	0.01
ttp	8	4.54	3.08	3.99

Experimental Results (extHyPre II)

- Impact of extHyPre preprocessing on bsolo

Benchmark	#	bsolo			
		w/o pre	extHypre (75,10,c)	extHypre (75,10,d)	extHypre (75,10,r)
aksoy	79	26	26	*25*	*23*
logic-synthesis	74	51	51	51	51
primes-dimacs-cnf	130	69	**71**	**70**	**70**
radar	12	6	6	6	6
routing	10	9	*8*	**10**	9
synthesis-ptl-cmos	8	6	6	6	6
testset	6	6	6	6	6
ttp	8	2	2	2	2
Total	327	175	**176**	**176**	*173*

Experimental Results (extHyPre II)

- Details on some instances of extHyPre preprocessing on bsolo:

Benchmark	Instance	bsolo			
		w/o pre	extHypre (75,10,c)	extHypre (75,10,d)	extHypre (75,10,r)
aksoy	fir07_area_delay	**16** ✔	**16** ✔	18	17
	fir09_area_opers	**34** ✔	**34** ✔	**34** ✔	36
	fir09_area_partials	**35** ✔	**35** ✔	**35** ✔	35
primes-dimacs-cnf	ii32b1	191	**191** ✔	191	**191** ✔
	ii32e4.opb	364	**364** ✔	**364** ✔	364
routing	s4-4-3-2pb	**64** ✔	66	**64** ✔	**64** ✔
	s4-4-3-3pb	68	72	**62** ✔	70

Experimental Results (extHyPre III)

- Impact of extHyPre preprocessing and Pueblo

Benchmark	#	Pueblo			
		w/o pre	extHypre (75,10,c)	extHypre (75,10,d)	extHypre (75,10,r)
aksoy	79	15	15	15	15
logic-synthesis	74	31	**32**	**32**	31
primes-dimacs-cnf	130	75	**78**	**77**	**78**
radar	12	0	0	0	0
routing	10	9	10	10	10
synthesis-ptl-cmos	8	6	6	6	6
testset	6	6	6	6	6
ttp	8	2	2	2	2
Total	327	140	**144**	**143**	**143**

Experimental Results (extHyPre III)

- Details on some instances of extHyPre preprocessing on Pueblo:

Benchmark	Instance	Pueblo			
		w/o pre	extHypre (75,10,c)	extHypre (75,10,d)	extHypre (75,10,r)
logic-synthesis	C880.a	**87** ✔	**87** ✔	**87** ✔	87
	sao2.b	25	**25** ✔	**25** ✔	**25** ✔
primes-dimacs-cnf	hanoi5	1931	**1931** ✔	**1931** ✔	**1931** ✔
	ii32c2	207	**207** ✔	**207** ✔	**207** ✔
	ii32e2	235	**235** ✔	235	**235** ✔

Experimental Results (extHyPre IV)

- Impact of extHyPre preprocessing on minisat+

Benchmark	#	minisat+			
		w/o pre	extHypre (75,10,c)	extHypre (75,10,d)	extHypre (75,10,r)
aksoy	79	24	24	**27**	**27**
logic-synthesis	74	31	*30*	*30*	*30*
primes-dimacs-cnf	130	79	**80**	**80**	79
radar	12	0	0	0	0
routing	10	10	10	10	10
synthesis-ptl-cmos	8	1	1	1	1
testset	6	4	4	4	4
ttp	8	2	2	2	2
Total	327	151	151	**154**	**153**

Experimental Results (extHyPre IV)

- Details on some instances of extHyPre preprocessing on minisat+:

Benchmark	Instance	minisat+			
		w/o pre	extHypre (75,10,c)	extHypre (75,10,d)	extHypre (75,10,r)
aksoy	fir05_area_delay	14	14	**14** ✔	**14** ✔
	matrix_5x3_5	21	21	**20** ✔	**20** ✔
	matrix_5x3_6	20	20	**20** ✔	**20** ✔
logic-synthesis	5xp1.b	**12** ✔	14	14	14
primes-dimacs-cnf	ii32d2	372	**372** ✔	**372** ✔	372

Conclusions & Future Work

- Applying SAT techniques in PBO preprocessing lead to a few improvements
- This take us to believe that further research can lead to better results
- In the future we plan to improve the efficiency of our PBO version of `extHyPre` and study different heuristics
- We also plan to incorporate other SAT techniques into PBO preprocessing