On Partitioning for Maximum Satisfiability

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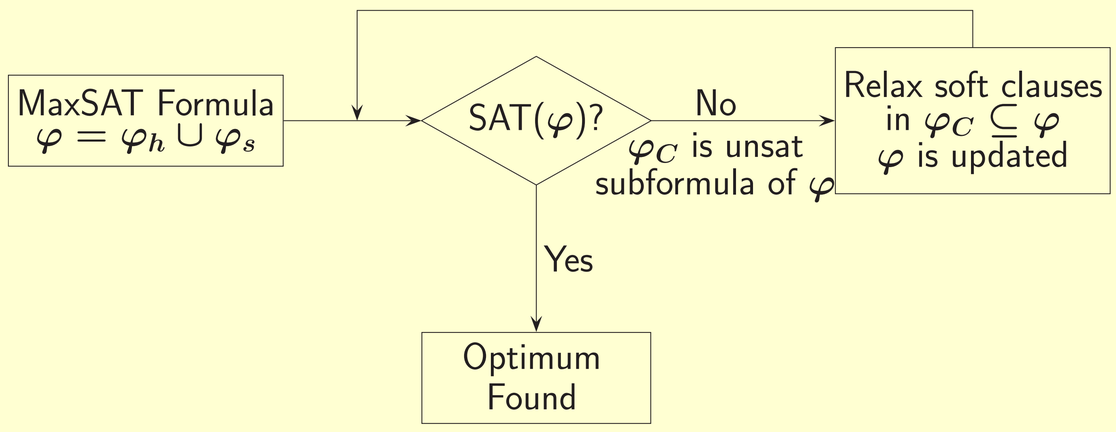
Maximum Satisfiability (MaxSAT)

- Optimization version of Boolean Satisfiability (SAT)
- Goal is to find an assignment to the variables such that it minimizes (maximizes) the number of unsatisfied (satisfied) clauses
- MaxSAT has several variants and can be generalized to the weighted partial MaxSAT problem. In this problem, some clauses are declared as hard, while the rest are declared as soft
- The goal in weighted partial MaxSAT is to find an assignment to the variables such that all hard clauses are satisfied, while minimizing the total weight of unsatisfied soft clauses

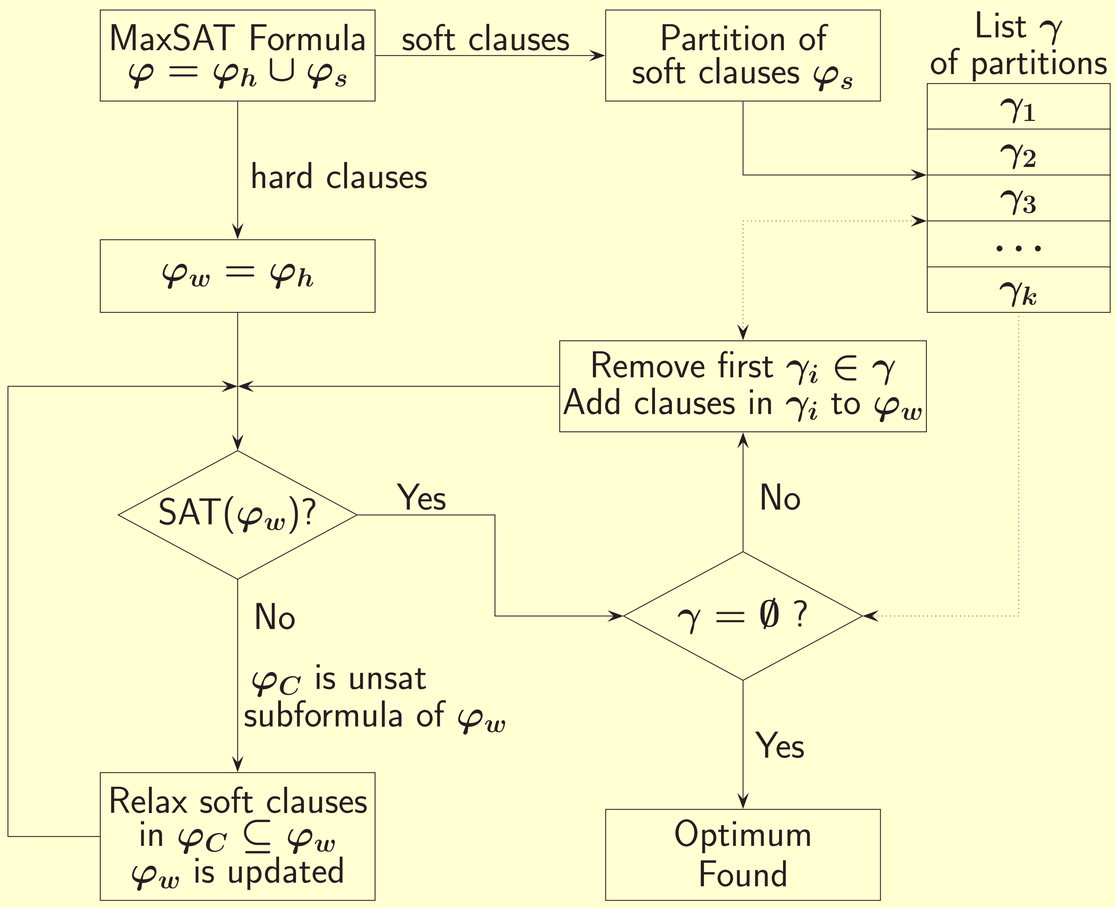
Basic Unsatisfiability-based Algorithms for MaxSAT

- Modern SAT solvers have the ability of producing certificates of unsatisfiability (identification of unsatisfiable subformulas)
- Start by considering all clauses (hard and soft) in the formula and iterate while the formula is unsatisfiable
- At each iteration, an unsatisfiable subformula is identified and relaxed
- The relaxation procedure consists of adding a new relaxation variable to each soft clause in the unsatisfiable subformula with a new constraint such that at most one of the new relaxation variables can be assigned value 1
- The algorithm stops when the working formula becomes satisfiable



Unsatisfiability-based algorithm with partitioning of soft clauses

- Split soft clauses into disjoint partitions
- Initial working formula only considers hard clauses
- If the working formula is unsatisfiable, apply relaxation to soft clauses in identified unsatisfiable subformula
- Whenever the working formula becomes satisfiable, add another partition of soft clauses into the working formula and continue
- The algorithm stops when the working formula is satisfiable and all partitions have been considered



Partitioning Soft Clauses

The motivation for partitioning is to focus the search on subsets of soft clauses, instead of considering all clauses at once.

Weight-based Partitioning

- Soft clauses with the same weight are likely to be related and are assigned to the same partition
- Partitions are sorted from the largest weight to the smallest
- Goal is to decrease the total number of iterations in the algorithm by finding first unsatisfiable subformulas with larger weights

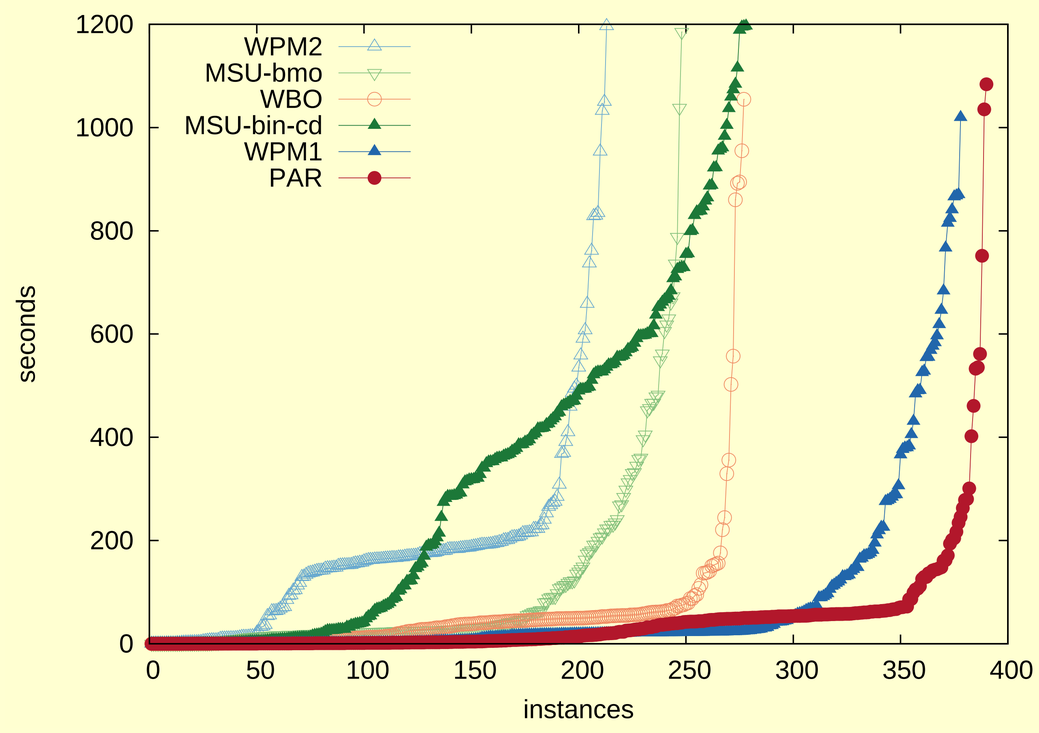
Graph-based Partitioning

- Goal is to find smaller unsatisfiable subformulas by taking advantage of localities in the MaxSAT instance
- A hypergraph is a generalization of a graph where an edge can connect any number of vertices
- Build a hypergraph from the weighted partial MaxSAT formula
- Soft and hard clauses of the formula are considered as vertices
- Each edge of the hypergraph represents a variable of the formula and connects all clauses (vertices) which contain that variable

Partition heuristic: use graph-based partitioning when the number of partitions in weight-based partitioning is large (> 300) and the average number of soft clauses in each partition is small (< 3).

Results: Number of instances solved by each MaxSAT solver

- Experiments were run on the weighted partial MaxSAT instances from the crafted and industrial categories of the MaxSAT evaluation of 2011
- Timeout of 1200 seconds
- Our new solver using soft clause partitioning (`PAR`) was built on top of `WBO`
- Compare with other unsatisfiability-based algorithms: `MSUncore` using lexicographical optimization (`MSU bmo`), `MSUncore` using core-guided binary search with disjoint cores (`MSU bin-cd`), `WPM1` and `WPM2`



- `PAR` clearly outperforms `WBO`
- On average, `WBO` performs 664 iterations, whereas `PAR` only needs 329 calls to the SAT solver
- `PAR` is the most robust solver as it solves the largest number of instances

Conclusions

- New unsatisfiability-based algorithm with partitioning of soft clauses
- Two different partition schemes are proposed
- Partitioning soft clauses can significantly improve the performance of unsatisfiability-based algorithms for weighted partial MaxSAT

Acknowledgments

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