Towards Smarter MACE-style Model Finders

Mikoláš Janota[1,2] Martin Suda[2]

[1] IST/INESC-ID, University of Lisbon, Portugal
[2] Czech Technical University in Prague

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Does a FOL formula have a finite model?

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- Finite models useful:
  - “debugging” of wrong theorems
  - “debugging” of wrong programs
  - information for lemma selection learning

Background and Motivation

TÉCNICO LISBOA

Does a FOL formula have a finite model?

- Finite models useful:
  - "debugging" of wrong theorems
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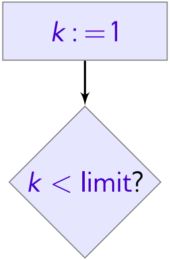
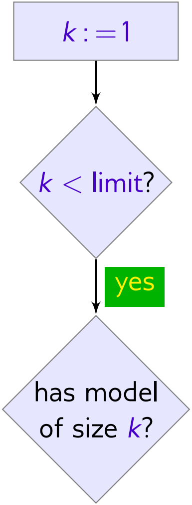
  - information for lemma selection learning

- **Advatage:**
  - If a finite model exists, it is fount in finite time.
  - Complete for some theories (Bernays-Schönfinkel, a.k.a. EPR)

MACE-style Framework



MACE-style Framework



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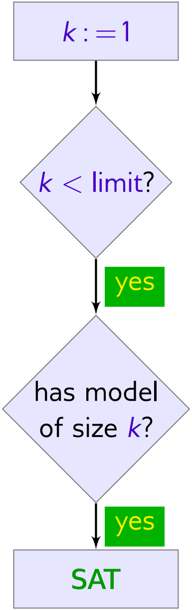

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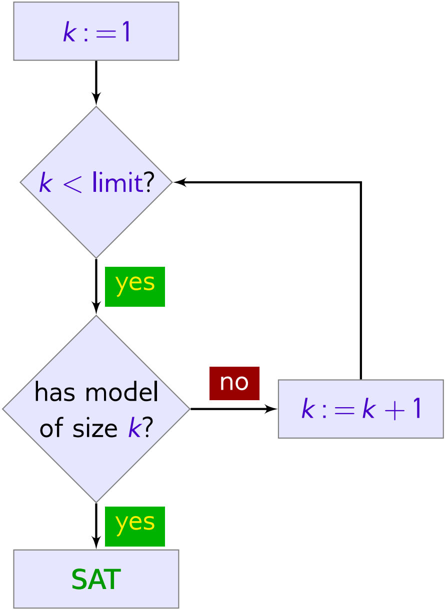

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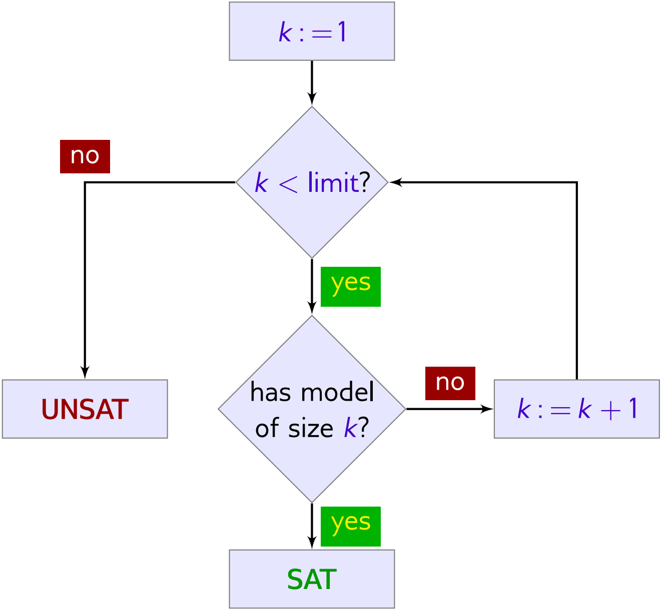

Avoiding Space Explosion

Issue:
Encoding directly to SAT is exponential, eventually blows up

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Remedy:
Encode into SAT **lazily** by
Count-**E**example **A**bstraction **G**uided **R**efinement (**CEGAR**)

$$(\exists \vec{p}\,\vec{f}\,)(\forall \vec{x}\,)\,\phi$$

$\vec{p}$ predicates, $\vec{f}$ functions, $\vec{x}$ FOL variables

Algorithm sketch:

1. $\alpha := \mathit{true}$

CEGAR for Fixed Size k

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7. **GOTO** 2

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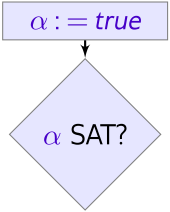

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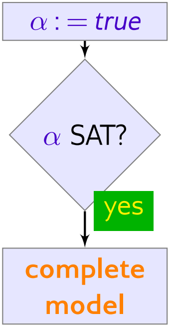

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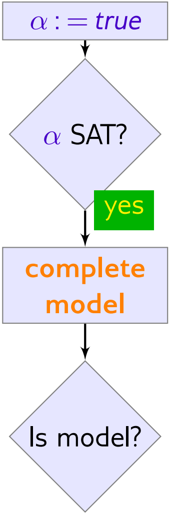

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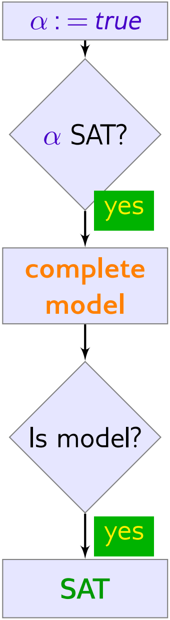

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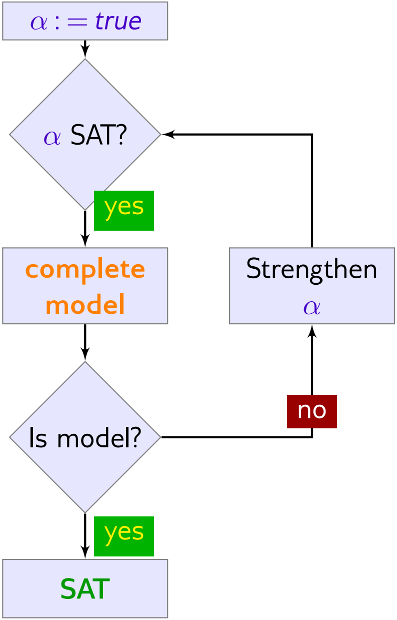

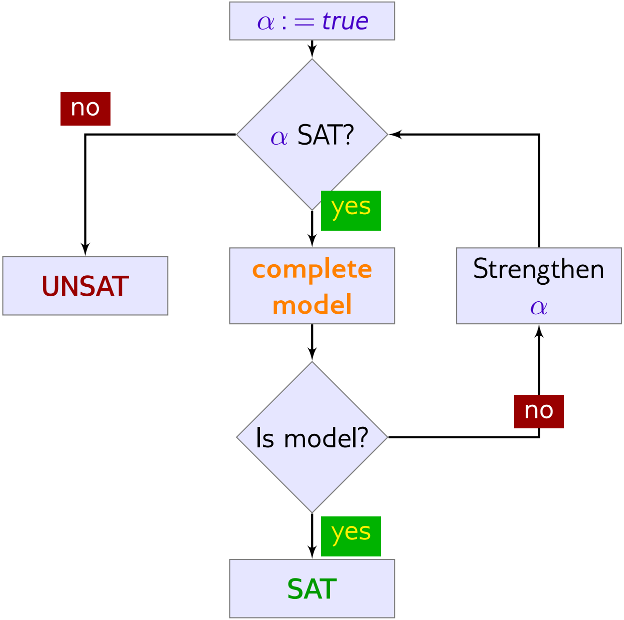
$\alpha := true$
α SAT?
no
UNSAT
yes
complete model
Strengthen α
Is model?
no
yes
SAT

We have model of ground α,

Example:

$$p(0), \neg q(0) \models p(0) \vee q(0)$$

Completing Models

We have model of ground α,
Example:

$$p(0), \neg q(0) \models p(0) \vee q(0)$$

We need to complete into **interpretation** of original
Examples for $(\forall x)(p(x) \vee \neg q(y))$

$$p \triangleq \{0\}, q \triangleq \{\}$$

$$p \triangleq \{0\}, q \triangleq \{1\}$$

$$p \triangleq 2^{1..k}, q \triangleq \{\}$$

Completing Models Contd.

Natural approach: set undefined to false/true

Examples

$$\{p(0), \neg p(1), \neg p(2)\} \;\ldots\; p \triangleq 2^{1..k} \setminus \{1, 2\}$$

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Remedy:
Learn the completion with Machine Learning techniques.

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5. Model of α:
   $t \triangleq 1$
   $p(0, \ldots, 0) \triangleq \text{False}$
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5. Model of α:
   $t \triangleq 1$
   $p(0,\ldots,0) \triangleq \text{False}$
   $p(1,\ldots,0) \triangleq \text{True}$
6. **Learn:**
   $t \triangleq 1$
   $p(x_1,\ldots,x_n) \triangleq (x_1 = 1)$

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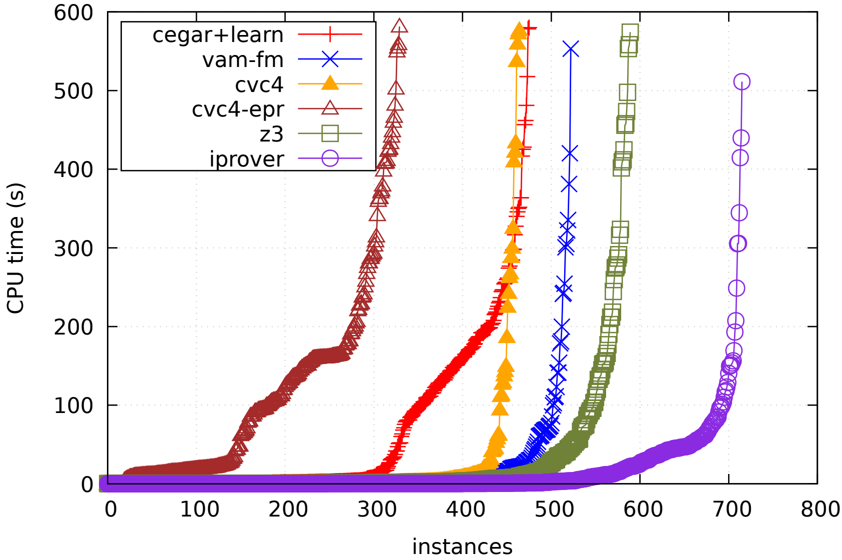
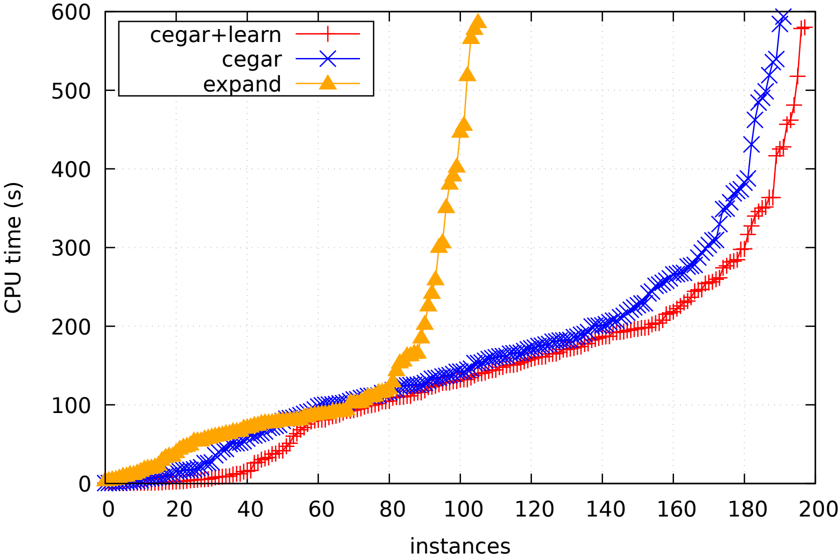
- Incremental SAT (`minisat`)
- Support for non-prenex
- Symmetry breaking, e.g. $c_1 \triangleq 0$

Results EPR

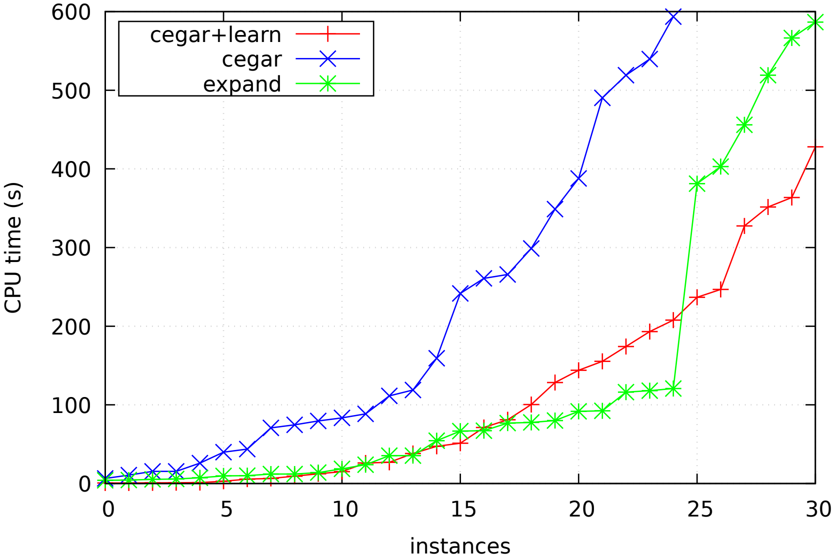


600
500
400
300
200
100
0
CPU time (s)
0
100
200
300
400
500
600
700
800
instances
cegar+learn
vam-fm
cvc4
cvc4-epr
z3
iprover

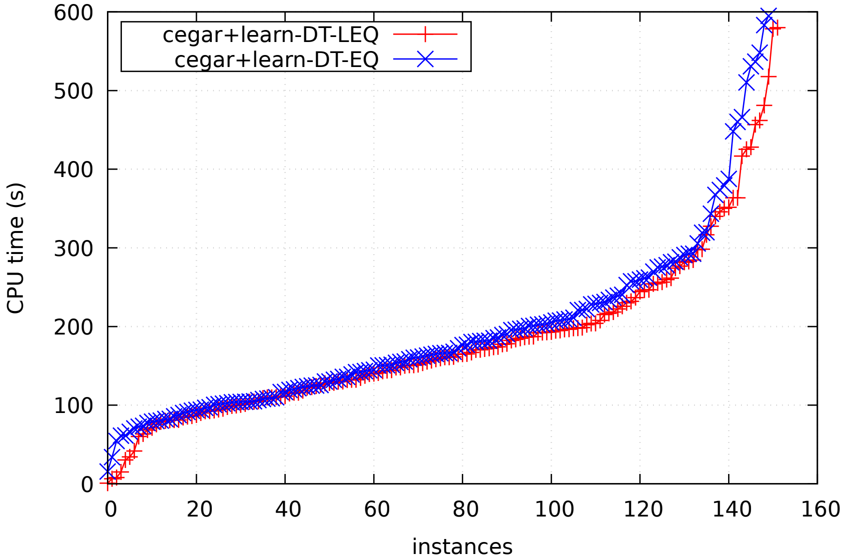
Results EPR: QFM



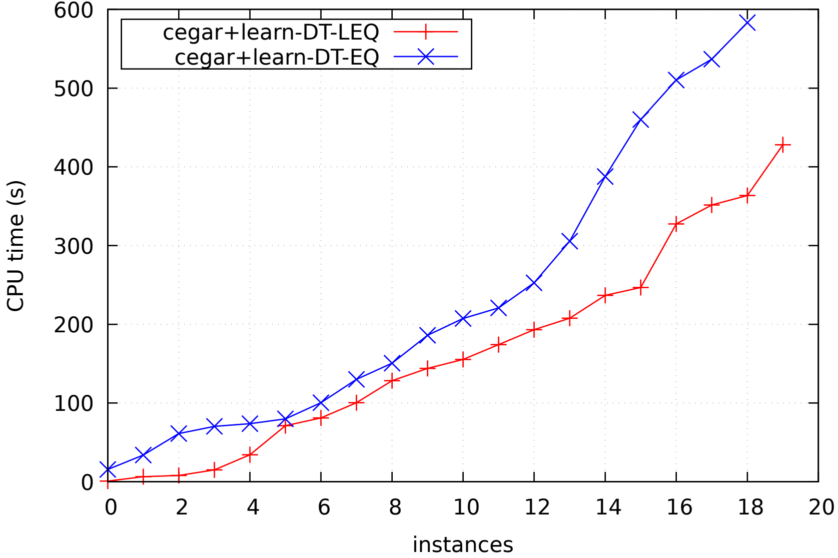
Results EPR QFM (SAT)



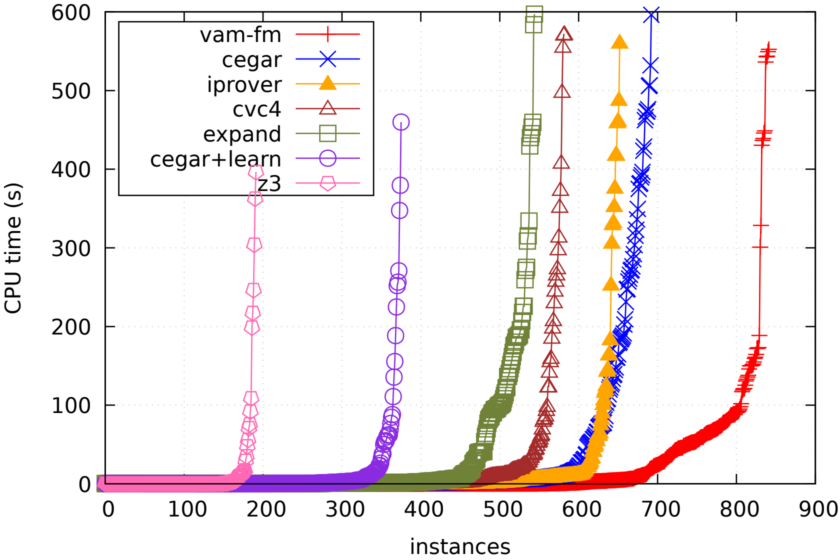
Results EPR: Learning Method



Results EPR: Learning Method (SAT)



Results SAT NON-EPR



Summary and Future

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- Learning in the presence of theories?
- Infinite domains?

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