Assertion-based Loop Invariant Generation

or, Counter-example Refinement Backwards

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Motivation

- we are looking for loop invariants that will show that something "bad" will not happen

```
for (int i = 0; i < a.length - 1; i++) {
    i = i + 1;
    a[i] = 0;
}
```

Motivation

- we are looking for loop invariants that will show that something "bad" will not happen

```
//@ loop_invariant a != null;
//@ loop_invariant i + 1 >= 0;
for (int i = 0; i < a.length - 1; i++) {
    i = i + 1;
    a[i] = 0;
}
```

Scary Slide

```
//@ requires a != null;
/*@ requires
  @  (\forall int x; (0 <= x & x < a.length) ==> a[x] != null); */
void setToZero(int [][] a) {
 /*@ loop_invariant
   @   (\forall int x; (0 <= x & x < a.length) ==> a[x] != null);
   @ loop_invariant a != null;
   @ loop_invariant i >= 0; */
 for (int i = 0; i < a.length; i++) {
  /*@ loop_invariant j >= 0;
    @ loop_invariant a != null;
    @ loop_invariant
    @   (\forall int x; (0 <= x & x < a.length) ==> a[x] != null); */
    for (int j = 0; j < a[i].length; j++)
      a[i][j] = 0;
  }
}
```

Invariants from Assertions Using Weakest Precondition

- let's assume that desired behavior is expressed as assertions
- using a weakest precondition calculus we can back-propagate assertions to the head of the loop

```
for (int i = 0; i < a.length - 1; i++) {
   i = i + 1;
   //@ assert a != null;
   //@ assert i <= 0;
   //@ assert i < a.length;
   a[i] = 0;
}
```

Invariants from Assertions Using Weakest Precondition

- let's assume that desired behavior is expressed as assertions
- using a weakest precondition calculus we can back-propagate assertions to the head of the loop

```
//@ loop_invariant (i < a.length - 1) ==> a != null;
```

```
for (int i = 0; i < a.length - 1; i++) {
   i = i + 1;
   //@ assert a != null;
   //@ assert i <= 0;
   //@ assert i < a.length;
   a[i] = 0;
}
```

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//@ loop_invariant (i < a.length - 1) ==> a != null;
//@ loop_invariant (i < a.length - 1) ==> 0 <= i + 1;
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```
for (int i = 0; i < a.length - 1; i++) {
  i = i + 1;
  //@ assert a != null;
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//@ loop_invariant (i < a.length - 1) ==> a != null;
//@ loop_invariant (i < a.length - 1) ==> 0 <= i + 1;
//@ loop_invariant (i < a.length - 1) ==> i + 1 < a.length;
for (int i = 0; i < a.length - 1; i++) {
  i = i + 1;
  //@ assert a != null;
  //@ assert i <= 0;
  //@ assert i < a.length;
  a[i] = 0;
}
```

Assertion Back-propagation

- we take a *nest* of loops

$$
\begin{array}{l}
P_0; \\
\{\mathcal{I}_1\}\ \textbf{while}\ (C_1)\ \textbf{do} \\
\quad \ldots \\
\qquad P_{n-1}; \\
\qquad \{\mathcal{I}_n\}\ \textbf{while}\ (C_n)\ \textbf{do} \\
\qquad\quad P_n; \\
\qquad\quad \textbf{assert}\ e;
\end{array}
$$

- and propagate the invariant outwards

$$
\begin{aligned}
\mathcal{I}_n &\equiv wlp(P_n, e) \\
\mathcal{I}_i &\equiv wlp(P_i, \mathcal{I}_{i+1})
\end{aligned}
$$

Are the Invariants OK?

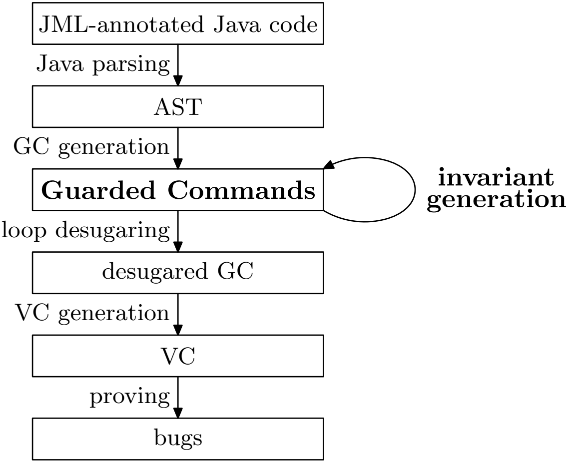
- all loops must *preserve* the pertaining invariant

$$\models \mathcal{J}_i \Rightarrow \mathit{wlp}(\text{LoopBody}_i, \mathcal{J}_i)$$

- the invariant of the outermost loop must be established by the preceding command

$$\models \mathit{wlp}(P_0, \mathcal{J}_1)$$

Implementation in ESC/Java2



Input Language

- Guarded Commands:

$$\mathit{cmd} := \quad x \leftarrow \mathit{expr} \mid \textbf{assume}\ f \mid \textbf{assert}\ f \mid \\ \mathit{cmd} \,[\!]\, \mathit{cmd} \mid \mathit{cmd}; \mathit{cmd} \mid \{\mathcal{I}\}\textbf{while}\ \mathit{expr}\ \textbf{do}\ \mathit{cmd}$$

  $\mathcal{I}, f$ are first-order logic formulas
- captures JML-annotated Java, examples:
  - **assert** pre — a precondition of a called method,
  - desired behavior, such as **assert** $a \neq$ null
  - **assume** post — a postcondition of a called method

Tweaking the Algorithm

- *invariant strengthening*, heuristically altering the invariant when found that it does not preserve the pertaining loop, e.g.,

$$\mathcal{I}[v \mapsto v'],\ v \text{ is free in } \mathcal{I} \text{ and } v' \text{ is a fresh variable}$$

- applying *formula simplifications* to the inferred invariants
- when computing weakest precondition, ignoring commands that do not seam *relevant* (relying on a heuristic)

Experiences and Future Work

- works splendidly on the examples I wrote, **nevertheless**,
- little of existing code is verifiable
- generated invariants are not too big, **nevertheless**,
- there is still opportunity to prune away trivial invariants
- reporting invariants to the user would serve as valuable feedback
- extending the analysis to take into account assertions **after** the loop would make the analysis considerably more useful