# On Minimal Corrections in ASP

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RCRA 2017, Bari

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- Literal a backbone if $\phi \models l$
  $\mathcal{L}_1 \subseteq \mathcal{L}_2 \Rightarrow \big(\bigwedge_{l \in \mathcal{L}_2} \phi \models l \Rightarrow \bigwedge_{l \in \mathcal{L}_1} \phi \models l\big)$

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\begin{array}{rcll}
 & \leftarrow & \mathit{not}\ \mathit{move}(a). & \texttt{\% program} \\
\mathit{move}(a) & \leftarrow & \mathit{stone}(b), \mathit{not}\ \mathit{stone}(c). & \texttt{\% program} \\
\mathit{stone}(c) & \leftarrow & . & \texttt{\% fact (input)}
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Possible fix: add $\mathit{stone}(b)$, remove $\mathit{stone}(c)$

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  - if the program $P \cup \{s. \mid s \in \mathcal{L}\}$ is consistent
  - and for any $\mathcal{L}'$, such that $\mathcal{L} \subsetneq \mathcal{L}' \subseteq \mathcal{S}$, the program $P \cup \{s. \mid s \in \mathcal{L}'\}$ is inconsistent.

**Observe:** *In monotone case $\mathcal{L}$ is maximally consistent iff $\mathcal{L} \cup \{s\}$ is inconsistent for any $s \in \mathcal{S} \setminus \mathcal{L}$. Does not hold in non-monotone.*

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## Idea

- Define $P' = P \cup \{s. \mid s \in \mathcal{L}\} \cup \{\texttt{choice}(\mathcal{S} \setminus \mathcal{L}).\}$.
- There exists a consistent set $\mathcal{L}'$ s.t. $\mathcal{L} \subseteq \mathcal{L}' \subseteq \mathcal{S}$ iff $P'$ has an answer set $\mu$ such that $\mathcal{L}' = \mathcal{S} \cap \mu$.

# Algorithm: At-least-1

$\mathcal{L} \leftarrow \emptyset$ // consistency lower bound

**while** true **do**

  $P' \leftarrow P \cup \{s. \mid s \in \mathcal{L}\}$

  $P' \leftarrow P' \cup \{\texttt{atleast1}(\mathcal{S} \setminus \mathcal{L}).\}$

  $(\text{res}, \mu) \leftarrow \texttt{solve}(P')$

  **if** $\neg$res **then return** $\mathcal{L}$

  $\mathcal{L} \leftarrow \mathcal{L} \cup (\mu \cap \mathcal{S})$

# Algorithm: Unit addition

$\mathcal{L} \leftarrow \emptyset$ // consistency lower bound
**while** $\mathcal{S} \neq \emptyset$ **do**
  $s_f \leftarrow$ pick an arbitrary element from $\mathcal{S}$
  $\mathcal{S} \leftarrow \mathcal{S} \setminus \{s_f\}$
  $\mathcal{L} \leftarrow \mathcal{L} \cup \{s_f\}$
  $P' \leftarrow P' \cup \{s. \mid s \in \mathcal{L}\}$
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  $(\text{res}, \mu) \leftarrow \texttt{solve}(P')$
  **if** $\neg\text{res}$ **then** $\mathcal{L} \leftarrow \mathcal{L} \setminus \{s_f\}$
  **else** $\mathcal{L} \leftarrow \mathcal{L} \cup (\mu \cap \mathcal{S})$
**return** $\mathcal{L}$

# Algorithm: Progression

$\mathcal{L} \leftarrow \emptyset$ `// consistency lower bound`
$K \leftarrow 1$ `// chunk size`
**while** $\mathcal{S} \neq \emptyset$ **do**
  $\mathcal{C} \leftarrow$ pick $\min(|\mathcal{S}|, K)$ arbitrary elements from $\mathcal{S}$
  $\mathcal{S} \leftarrow \mathcal{S} \setminus \mathcal{C}$
  $\mathcal{L} \leftarrow \mathcal{L} \cup \mathcal{C}$
  $P' \leftarrow P' \cup \{s. \mid s \in \mathcal{L}\}$
  $P' \leftarrow P \cup \{\texttt{choice}(\mathcal{S}).\}$
  $(\text{res}, \mu) \leftarrow \texttt{solve}(P')$
  **if** $\neg\text{res}$ **then** `// re-analyze chunk more finely`
    $\mathcal{L} \leftarrow \mathcal{L} \setminus \mathcal{C}$
    **if** $K > 1$ **then** $\mathcal{S} \leftarrow \mathcal{S} \cup \mathcal{C}$
    $K = 1$ `// reset chunk size`
  **else**
    $K \leftarrow 2K$ `// double chunk size`
    $\mathcal{L} \leftarrow \mathcal{L} \cup (\mu \cap \mathcal{S})$
**return** $\mathcal{L}$

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    - $(P \setminus M_r) \cup M_a$ is consistent.
- An $(\mathcal{A}, \mathcal{R})$-correction $(M_r, M_a)$ is minimal if for any $(\mathcal{A}, \mathcal{R})$-correction $(M'_r, M'_a)$ such that $M'_r \subseteq M_r$ and $M'_a \subseteq M_a$, it holds that $M_a = M'_a$ and $M_r = M'_r$.

# Minimal Correction Sets and Maximal Consistency

To calculate $(\mathcal{A}, \mathcal{R})$-correction via Maximal Consistency:

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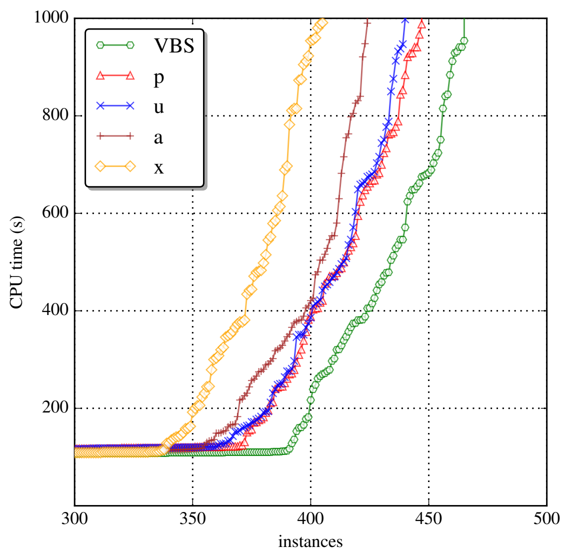
- Maximal consistent subset of the fresh atoms gives a minimal correction.

# Experimental Results

| Family | a | p | u | x | VBS |
|---|---|---|---|---|---|
| knight [8,10] (95) | 74 | 75 | **78** | 60 | 80 |
| knight [8,4] (51) | 7 | **13** | **13** | 7 | 14 |
| patterns [16,10] (100) | **100** | **100** | **100** | **100** | 100 |
| patterns [20,15] (100) | **100** | **100** | **100** | **100** | 100 |
| solitaire [12] (18) | **18** | **18** | **18** | 17 | 18 |
| solitaire [14] (16) | **12** | 9 | 11 | 4 | 13 |
| graceful graphs [10,50] (100) | 57 | **75** | 63 | 62 | 83 |
| graceful graphs [30,20] (57) | 56 | **57** | **57** | 55 | 57 |
| *total (537)* | 424 | **447** | 440 | 405 | 465 |

# Experimental Results (Cont.)



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- More algorithms?
- How to obtain the “addition set”?
- What are the good means for users to specify the addition and removal sets?

Thank You for Your Attention!

Questions?

Marques-Silva, J., Janota, M., and Belov, A. (2013).
**Minimal sets over monotone predicates in boolean formulae.**
In *Computer Aided Verification - International Conference (CAV)*, pages 592–607.